24. The Circle

Exercise 24.1

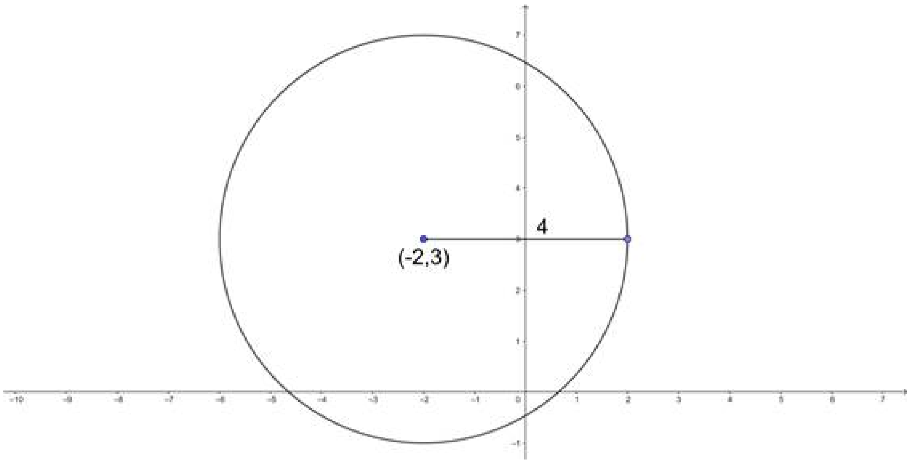
1 A. Question

Find the equation of the circle with:

Centre (- 2, 3) and radius 4.

Answer

(i) Given that we need to find the equation of the circle with centre (- 2, 3) and radius 4.



We know that the equation of the circle with centre (p, q) and radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x - (- 2))^2 + (y - 3)^2 = 4^2$

⇒ $(x + 2)^2 + (y - 3)^2 = 16$

⇒ $x^2 + 4x + 4 + y^2 - 6y + 9 = 16$

⇒ $x^2 + y^2 + 4x - 6y - 3 = 0$

∴The equation of the circle is $x^2 + y^2 + 4x - 6y - 3 = 0$.

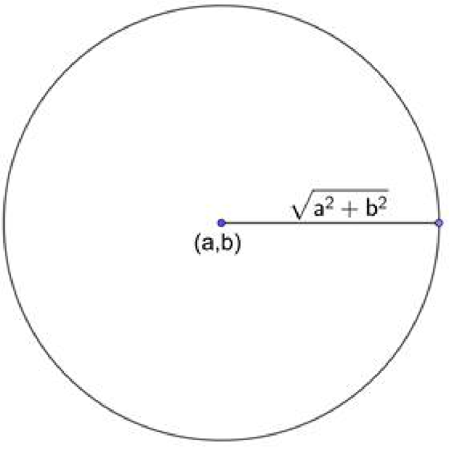
1 B. Question

Find the equation of the circle with:

Centre (a, b) and radius $\sqrt{a^2 + b^2}$.

Answer

Given that we need to find the equation of the circle with centre (a, b) and radius $\sqrt{a^2 + b^2}$.



We know that the equation of the circle with centre (p, q) and radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - a)^2 + (y - b)^2 = \left(\sqrt{a^2 + b^2}\right)^2$

$\Rightarrow x^2 - 2ax + a^2 + y^2 - 2by + b^2 = a^2 + b^2$

$\Rightarrow x^2 + y^2 - 2ax - 2by = 0$

$\therefore$The equation of the circle is $x^2 + y^2 - 2ax - 2by = 0$.

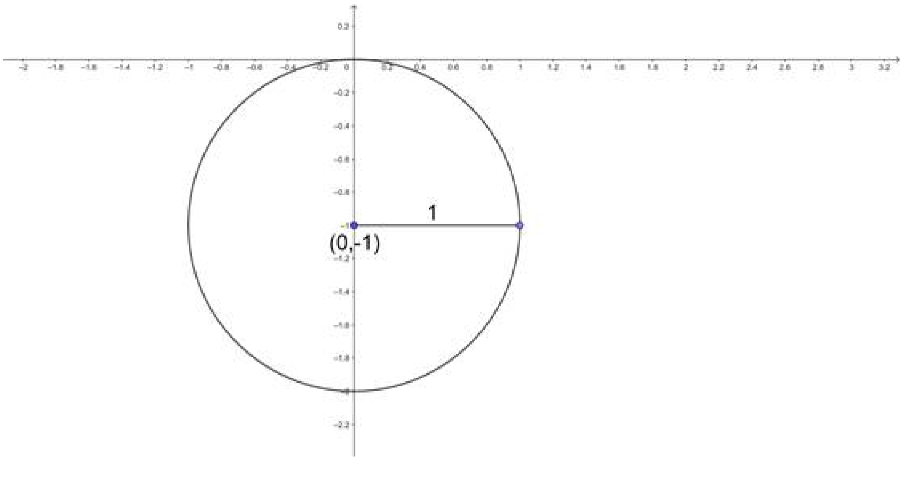
1 C. Question

Find the equation of the circle with:

Centre (0, - 1) and radius 1.

Answer

Given that we need to find the equation of the circle with centre (0, - 1) and radius 1.



We know that the equation of the circle with centre (p, q) and radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - 0)^2 + (y - (- 1))^2 = 1^2$

$\Rightarrow (x - 0)^2 + (y + 1)^2 = 1$

$\Rightarrow x^2 + y^2 + 2y + 1 = 1$

$\Rightarrow x^2 + y^2 + 2y = 0$

∴The equation of the circle is $x^2 + y^2 + 2y = 0$.

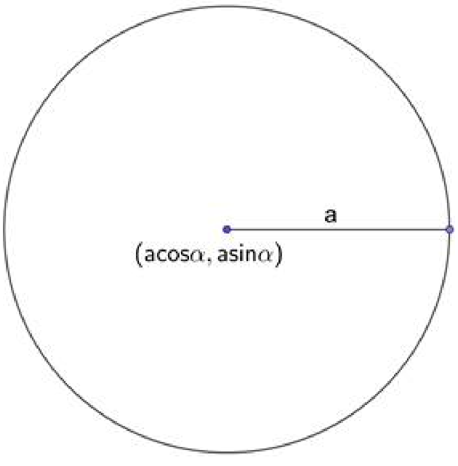
1 D. Question

Find the equation of the circle with:

Centre (a cosα, a sinα) and radius a.

Answer

Given that we need to find the equation of the circle with centre (acosα, asinα) and radius a.



We know that the equation of the circle with centre (p, q) and radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - a\cos\alpha)^2 + (y - a\sin\alpha)^2 = a^2$

$\Rightarrow x^2 - (2a\cos\alpha)x + a^2\cos^2\alpha + y^2 - (2a\sin\alpha)y + a^2\sin^2\alpha = a^2$

We know that $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow x^2 - (2a\cos\alpha)x + y^2 - 2a\sin\alpha y + a^2 = a^2$

$\Rightarrow x^2 + y^2 - (2a\cos\alpha)x - (2a\sin\alpha)y = 0$

∴The equation of the circle is $x^2 + y^2 - (2a\cos\alpha)x - (2a\sin\alpha)y = 0$.

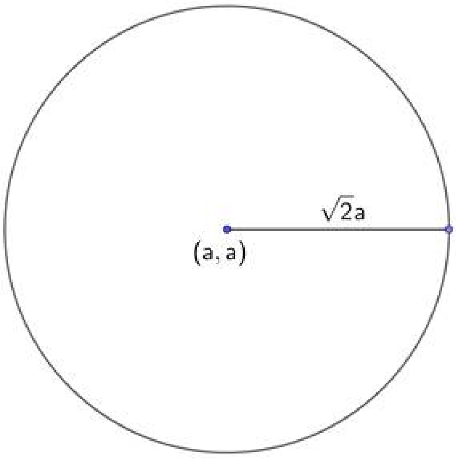
1 E. Question

Find the equation of the circle with:

Centre (a, a) and radius √2 a.

Answer

Given that we need to find the equation of the circle with centre (a, a) and radius√2a.



We know that the equation of the circle with centre (p, q) and radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x - a)^2 + (y - a)^2 = (\sqrt{2}a)^2$

⇒ $x^2 - 2ax + a^2 + y^2 - 2ay + a^2 = 2a^2$

⇒ $x^2 + y^2 - 2ax - 2ay = 0$

∴The equation of the circle is $x^2 + y^2 - 2ax - 2ay = 0$.

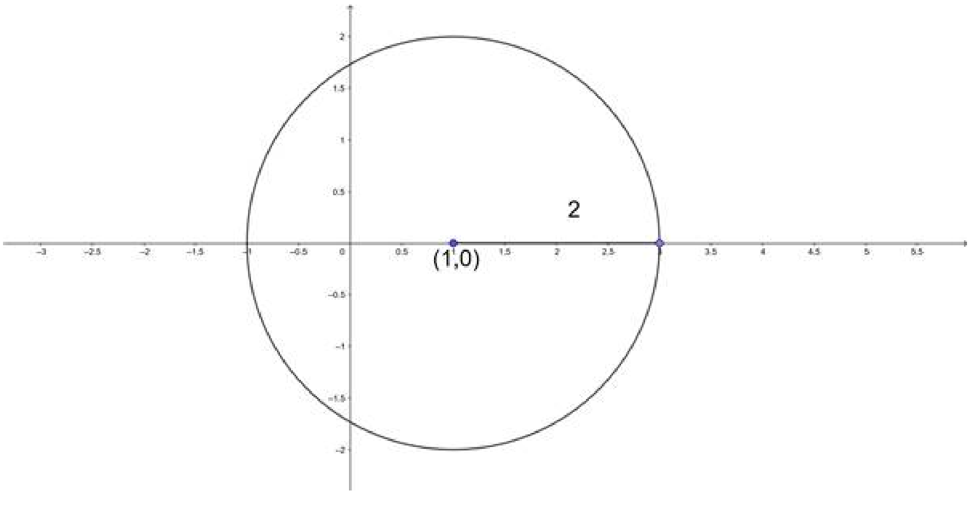
2 A. Question

Find the centre and radius of each of the following circles:

$(x - 1)^2 + y^2 = 4$

Answer

(i) Given that we need to find the centre and radius of the given circle $(x - 1)^2 + y^2 = 4$.



We know that the standard equation of a circle with centre (a, b) and having radius ‘r’ is given by:

⇒ $(x - a)^2 + (y - b)^2 = r^2$ - - - - - (1)

Let us convert given circle’s equation into the standard form.

⇒ $(x - 1)^2 + y^2 = 4$

⇒ $(x - 1)^2 + (y - 0)^2 = 2^2$ ..... - (2)

Comparing (2) with (1), we get

⇒ Centre = (1, 0) and radius = 2

∴The centre and radius of the circle is (1, 0) and 2.

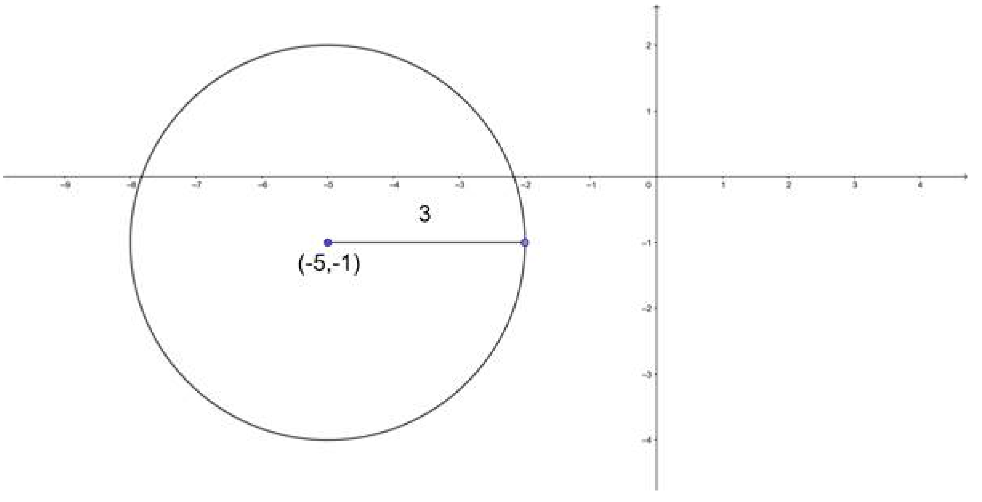
2 B. Question

Find the centre and radius of each of the following circles:

$(x + 5)^2 + (y + 1)^2 = 9$

Answer

Given that we need to find the centre and radius of the given circle $(x + 5)^2 + (y + 1)^2 = 9$.



We know that the standard equation of a circle with centre (a, b) and having radius ‘r’ is given by:

$\Rightarrow (x - a)^2 + (y - b)^2 = r^2$ - - - - - (1)

Let us convert given circle’s equation into the standard form.

$\Rightarrow (x + 5)^2 + (y + 1)^2 = 9$

$\Rightarrow (x - (-5))^2 + (y - (-1))^2 = 3^2$ - - - - (2)

Comparing (2) with (1), we get

⇒ Centre = (- 5, - 1) and radius = 3

∴The centre and radius of the circle is (- 5, - 1) and 3.

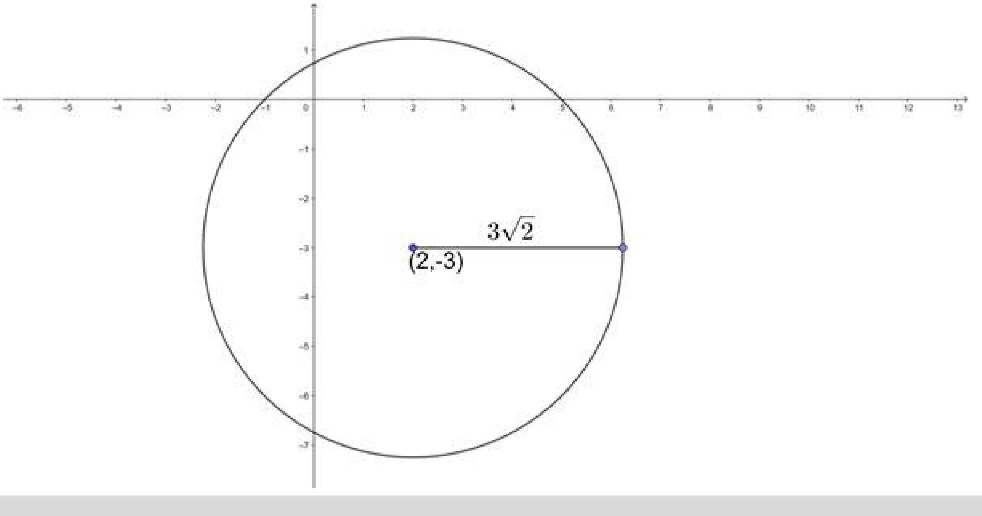
2 C. Question

Find the centre and radius of each of the following circles:

$x^2 + y^2 - 4x + 6y = 5$

Answer

Given that we need to find the centre and radius of the given circle $x^2 + y^2 - 4x + 6y = 5$.



We know that the standard equation of a circle with centre (a, b) and having radius ‘r’ is given by:

⇒ $(x - a)^2 + (y - b)^2 = r^2$ - - - - - (1)

Let us convert given circle’s equation into the standard form.

⇒ $x^2 + y^2 - 4x + 6y = 5$

⇒ $(x^2 - 4x + 4) + (y^2 + 6y + 9) = 5 + 4 + 9$

⇒ $(x - 2)^2 + (y + 3)^2 = 18$

⇒ $(x - 2)^2 + (y - (- 3))^2 = (3\sqrt{2})^2$ - - - (2)

Comparing (2) with (1), we get

⇒ Centre = (2, - 3) and radius = $3\sqrt{2}$

∴The centre and radius of the circle is (2, - 3) and $3\sqrt{2}$.

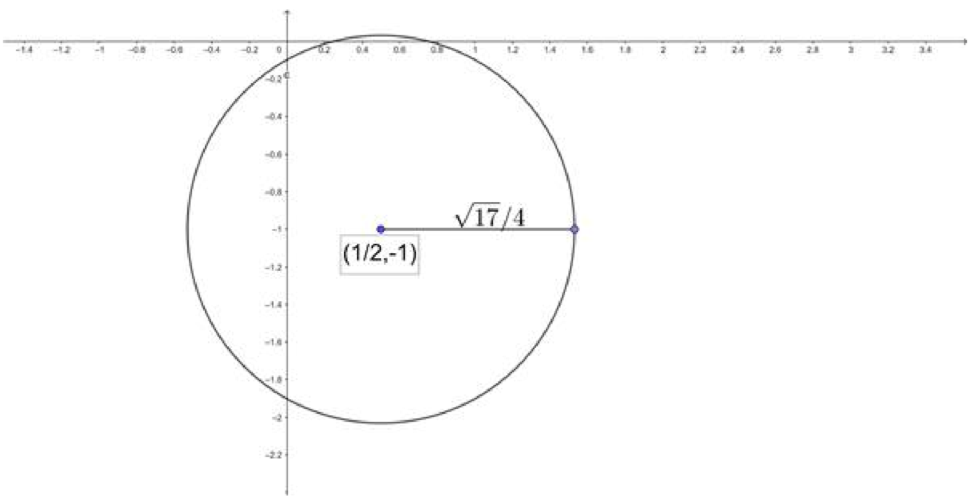
2 D. Question

Find the centre and radius of each of the following circles:

$x^2 + y^2 - x + 2y - 3 = 0$

Answer

Given that we need to find the centre and radius of the given circle $x^2 + y^2 - x + 2y - 3 = 0$.



We know that the standard equation of a circle with centre (a, b) and having radius ‘r’ is given by:

⇒ $(x - a)^2 + (y - b)^2 = r^2$ - - - - - (1)

Let us convert given circle’s equation into the standard form.

⇒ $x^2 + y^2 - x + 2y - 3 = 0$

⇒ $\left(x^2 - x + \frac{1}{4}\right) + (y^2 + 2y + 1) - 3 - \frac{1}{4} - 1 = 0$

⇒ $\left(x - \frac{1}{2}\right)^2 + (y + 1)^2 = \frac{17}{4}$

⇒ $\left(x - \frac{1}{2}\right)^2 + (y + 1)^2 = \left(\frac{\sqrt{17}}{2}\right)^2$ - - - - - (2)

Comparing (2) with (1), we get

⇒ Centre = $(\frac{1}{2}, - 1)$ and radius = $\frac{\sqrt{17}}{2}$

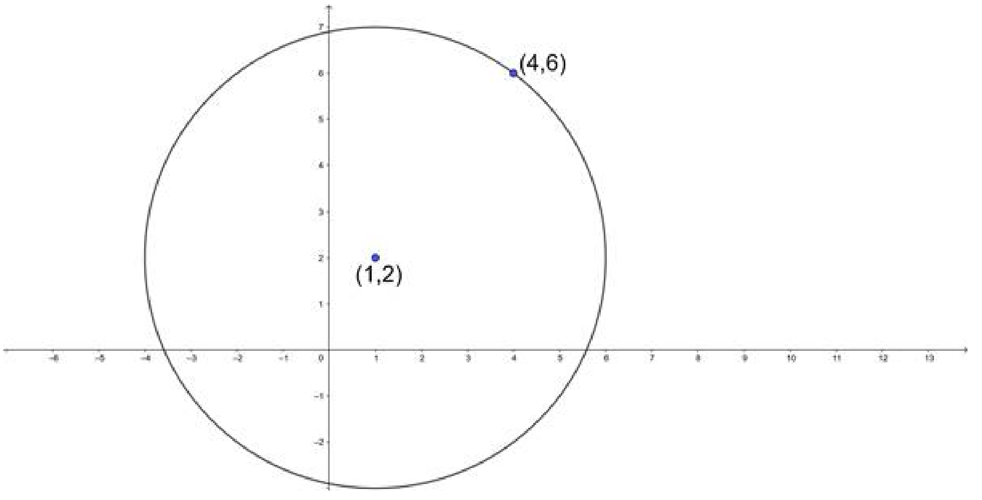
∴The centre and radius of the circle is $(\frac{1}{2}, -1)$ and $\frac{\sqrt{17}}{2}$.

3. Question

Find the equation of the circle whose centre is (1, 2) and which passes through the point (4, 6).

Answer

Given that we need to find the equation of the circle whose centre is (1, 2) and passing through the point (4, 6).



We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(1-4)^2 + (2-6)^2}$

$\Rightarrow r = \sqrt{(-3)^2 + (-4)^2}$

$\Rightarrow r = \sqrt{9 + 16}$

$\Rightarrow r = \sqrt{25}$

⇒ r = 5 units ..... (1)

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - 1)^2 + (y - 2)^2 = 5^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 = 25$

$\Rightarrow x^2 + y^2 - 2x - 4y - 20 = 0.$

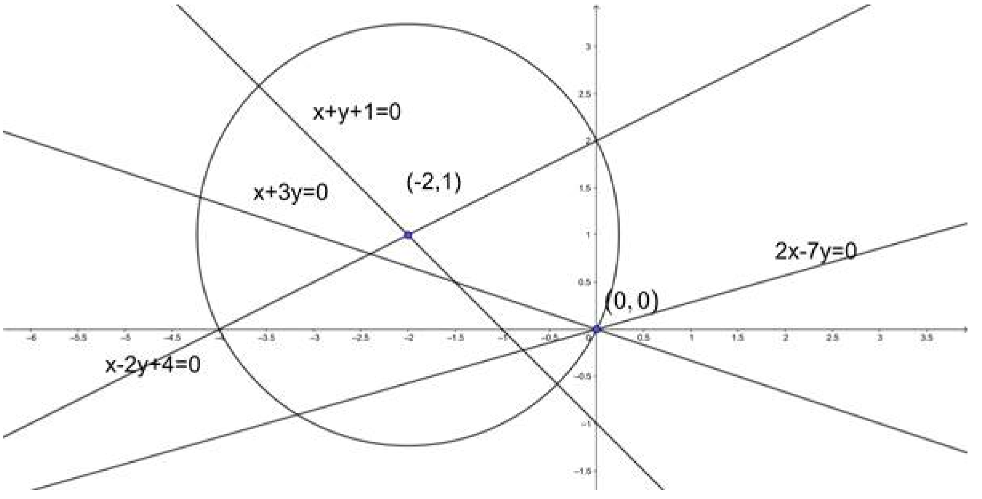
∴The equation of the circle is $x^2 + y^2 - 2x - 4y - 20 = 0$.

4. Question

Find the equation of the circle passing through the point of intersection of the lines x + 3y = 0 and 2x – 7y = 0 and whose centre is the point of intersection of the lines x + y + 1 = 0 and x – 2y + 4 = 0.

Answer

Given that the circle has the centre at the intersection point of the lines x + y + 1 = 0 and x - 2y + 4 = 0 and passes through the point of intersection of the lines x + 3y = 0 and 2x - 7y = 0.



Let us find the points of intersection of the lines.

On solving the lines x + 3y = 0 and 2x - 7y = 0, we get the point of intersection to be (0, 0)

On solving the lines x + y + 1 and x - 2y + 4 = 0, we get the point of intersection to be (- 2, 1)

We have circle with centre (- 2,1) and passing through the point (0,0).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(-2 - 0)^2 + (1 - 0)^2}$

$\Rightarrow r = \sqrt{(-2)^2 + (1)^2}$

$\Rightarrow r = \sqrt{4 + 1}$

$\Rightarrow r = \sqrt{5}$ ..... (1)

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - (-2))^2 + (y - 1)^2 = (\sqrt{5})^2$

$\Rightarrow (x + 2)^2 + (y - 1)^2 = 5$

$\Rightarrow x^2 + 4x + 4 + y^2 - 2y + 1 = 5$

$\Rightarrow x^2 + y^2 + 4x - 2y = 0.$

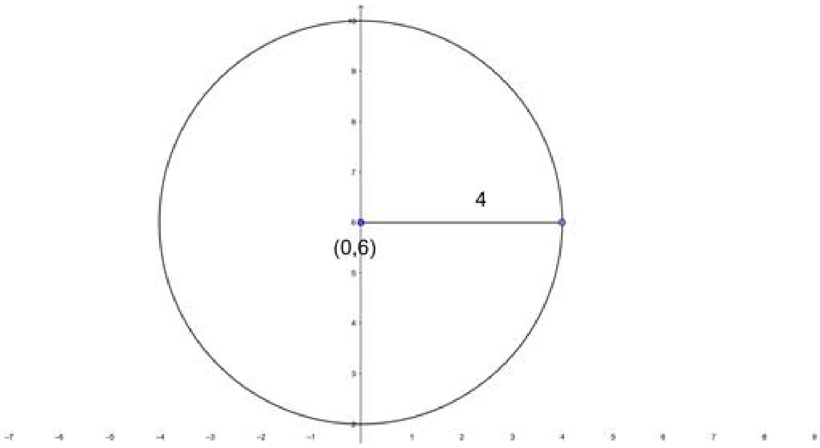
∴The equation of the circle is $x^2 + y^2 + 4x - 2y = 0$.

5. Question

Find the equation of the circle whose centre lies on the positive direction of y - axis at a distance 6 from the origin and whose radius is 4.

Answer

Given that we need to find the equation of the circle whose centre lies on the positive y - axis at a distance of 6 from the origin and having radius 4.



Since the centre lies on the positive y - axis at a distance of 6 from the origin, we get the centre (0, 6).

We have a circle with centre (0, 6) and having radius 4.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - 0)^2 + (y - 6)^2 = 4^2$

$\Rightarrow x^2 + y^2 - 12y + 36 = 16$

$\Rightarrow x^2 + y^2 - 12y + 20 = 0.$

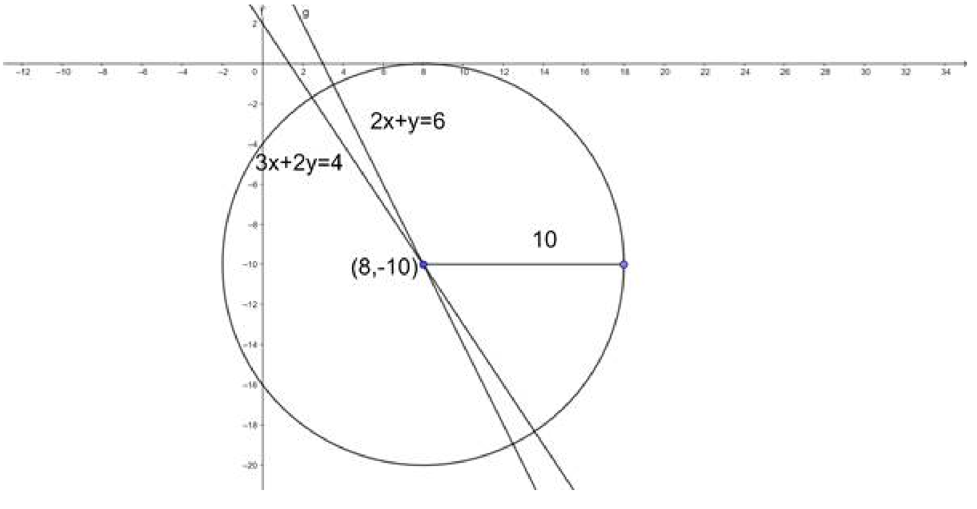
$\therefore$The equation of the circle is $x^2 + y^2 - 12y + 20 = 0$.

6. Question

If the equations of two diameters of a circle are 2x + y = 6 and 3x + 2y = 4 and the radius is 10, find the equation of the circle.

Answer

Given that the circle has the radius 10 and has diameters 2x + y = 6 and 3x + 2y = 4.



We know that the centre is the intersection point of the diameters.

On solving the diameters, we get the centre to be (8, - 10).

We have a circle with centre (8, - 10) and having radius 10.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x - 8)^2 + (y - (-10))^2 = 10^2$

⇒ $(x - 8)^2 + (y + 10)^2 = 100$

⇒ $x^2 - 16x + 64 + y^2 + 20y + 100 = 100$

⇒ $x^2 + y^2 - 16x + 20y + 64 = 0.$

∴The equation of the circle is $x^2 + y^2 - 16x + 20y + 64 = 0.$

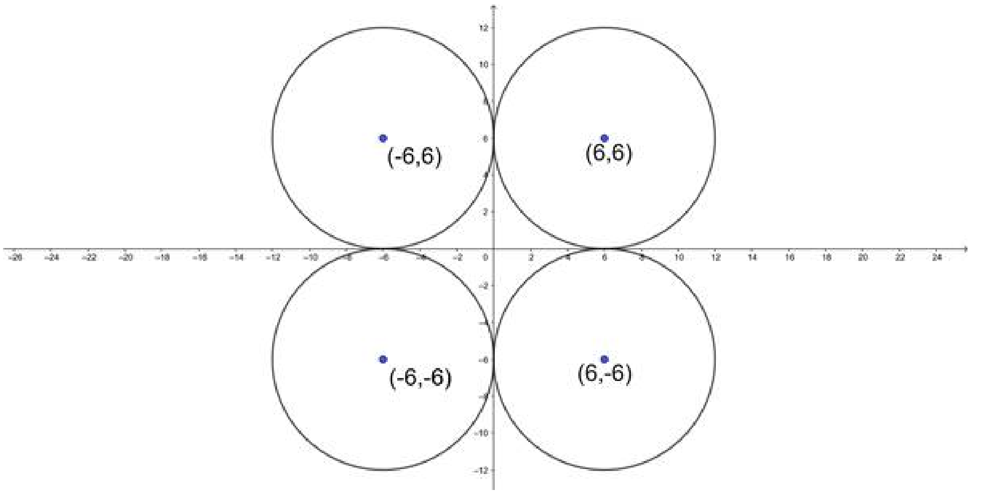
7 A. Question

Find the equation of the circle

Which touches both the axes at a distance of 6 units from the origin.

Answer

Given that we need to find the equation of the circle which touches both the axes at a distance of 6 units from the origin.



A circle touches the axes at the points (±6, 0) and (0,±6).

From the figure, we can see that the centre of the circle is (±6,±6).

We have a circle with centre (±6,±6) and passing through the point (0,6).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

⇒ $r = \sqrt{(6-0)^2 + (6-6)^2}$

⇒ $r = \sqrt{(6)^2 + (0)^2}$

⇒ $r = \sqrt{36}$

⇒ r = 6 ..... (1)

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x \pm 6)^2 + (y \pm 6)^2 = (6)^2$

⇒ $x^2 \pm 12x + 36 + y^2 \pm 12y + 36 = 36$

$\Rightarrow x^2 + y^2 \pm 12x \pm 12y + 36 = 0.$

$\therefore$The equation of the circle is $x^2 + y^2 \pm 12x \pm 12y + 36 = 0$. We get four circles which satisfy this condition.

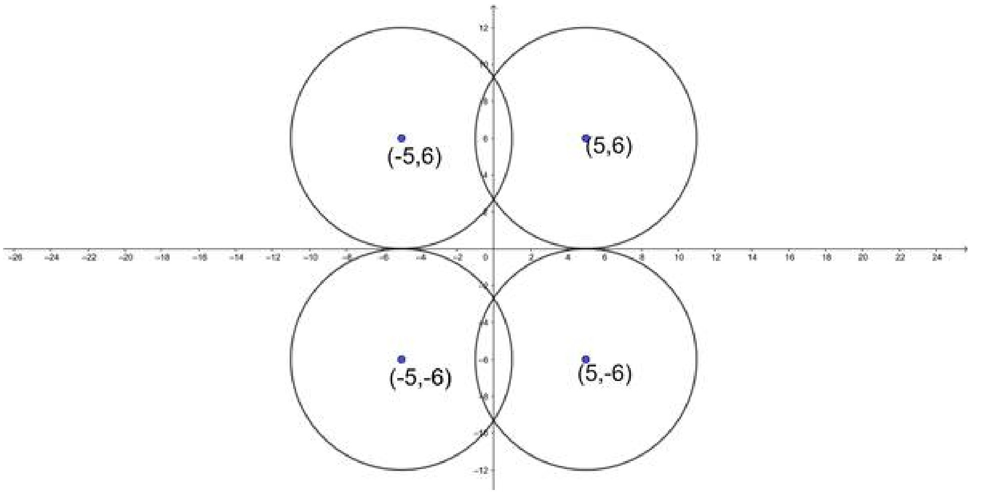
7 B. Question

Find the equation of the circle

Which touches x - axis at a distance of 5 from the origin and radius 6 units.

Answer

Given that we need to find the equation of the circle which touches x - axis at a distance of 5 units from the origin and having radius 6 units.



A circle touches the x - axis at the points (±5, 0).

Let us assume the centre of the circle is (±5, a).

We have a circle with centre (5, a) and passing through the point (5,0) and having radius 6.

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow 6 = \sqrt{(5-5)^2 + (a-0)^2}$

$\Rightarrow 6 = \sqrt{(0)^2 + (a)^2}$

$\Rightarrow 6 = \sqrt{(a^2)}$

$\Rightarrow |a| = 6$

$\Rightarrow a = \pm 6$ ..... (1)

We have got the centre at (±5, ±6) and having radius 6 units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x \pm 5)^2 + (y \pm 6)^2 = (6)^2$

$\Rightarrow x^2 \pm 10x + 25 + y^2 \pm 12y + 36 = 36$

$\Rightarrow x^2 + y^2 \pm 10x \pm 12y + 25 = 0.$

$\therefore$The equation of the circle is $x^2 + y^2 \pm 10x \pm 12y + 25 = 0$. We get four circles which satisfy this condition.

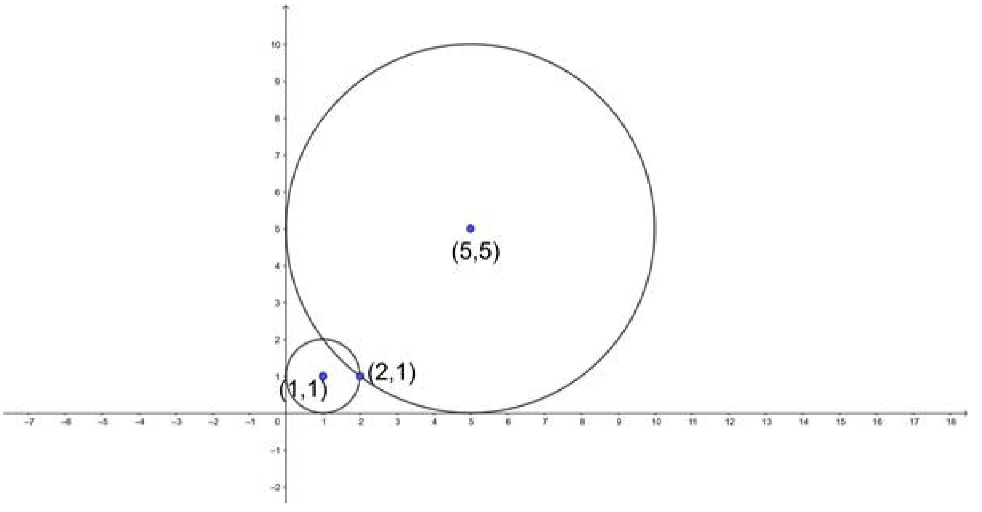
7 C. Question

Find the equation of the circle

Which touches both the axes and passes through the point (2, 1)

Answer

Given that we need to find the equation of the circle which touches both axes and passes through the point (2, 1).



Let us assume the circle touches the x-axis at the point (a, 0) and y-axis at the point (0,a).

Then we get to the centre of the circle as (a, a).

We have a circle with centre (a, a) and passing through the point (2, 1) and having radius |a|.

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow |a| = \sqrt{(a-2)^2+(a-1)^2}$

$\Rightarrow a^2 = a^2 - 4a + 4 + a^2 - 2a + 1$

$\Rightarrow a^2 - 6a + 5 = 0$

$\Rightarrow a^2 - 5a - a + 5 = 0$

$\Rightarrow a(a - 5) - 1(a - 5) = 0$

$\Rightarrow (a - 1)(a - 5) = 0$

$\Rightarrow a - 1 = 0$ or $a - 5 = 0$

$\Rightarrow a = 1$ or $a = 5$ ..... (1)

Case (i)

We have got the centre at (5, 5) and having radius 5 units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - 5)^2 + (y - 5)^2 = 5^2$

$\Rightarrow x^2 - 10x + 25 + y^2 - 10y + 25 = 25$

$\Rightarrow x^2 + y^2 - 10x - 10y + 25 = 0.$

∴The equation of the circle is $x^2 + y^2 - 10x - 10y + 25 = 0.$

Case (ii)

We have got the centre at (1, 1) and having a radius 1 unit.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - 1)^2 + (y - 1)^2 = 1^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 2y + 1 = 1$

$\Rightarrow x^2 + y^2 - 2x - 2y + 1 = 0$

∴ The equation of the circle is $x^2 + y^2 - 2x - 2y + 1 = 0.$

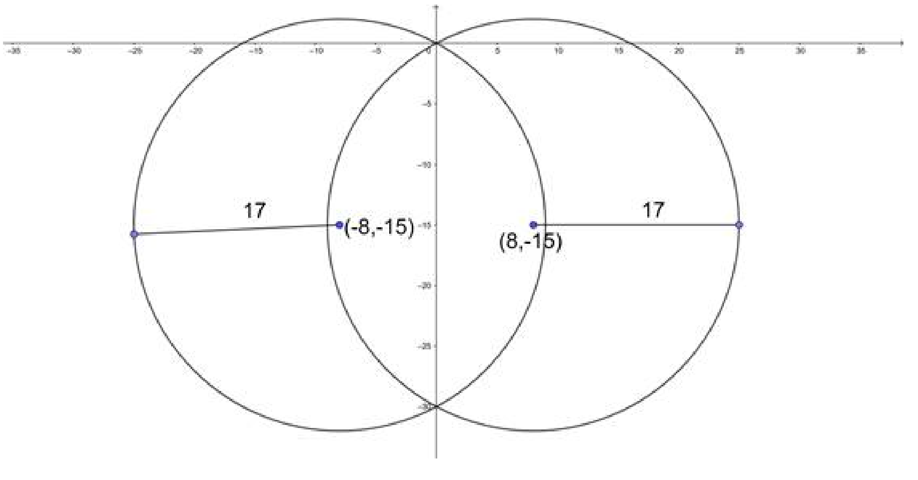
7 D. Question

Find the equation of the circle

Passing through the origin, radius 17 and ordinate of the centre is - 15.

Answer

Given that we need to find the equation of the circle which passes through the origin, having radius 17 units and ordinate of the centre is - 15.



Let us assume the abscissa of the centre be a then we get to the centre of the circle as (a, - 15).

We have a circle with centre (a, - 15) and passing through the point (0, 0) and having radius 17.

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow 17 = \sqrt{(a - 0)^2 + (-15 - 0)^2}$

$\Rightarrow 17^2 = a^2 + (-15)^2$

$\Rightarrow 289 = a^2 + 225$

$\Rightarrow a^2 = 64$

$\Rightarrow |a| = \sqrt{64}$

⇒ |a| = 8

⇒ a = ±8 ..... (1)

We have got the centre at (±8, - 15) and having radius 17 units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x \pm 8)^2 + (y - 15)^2 = 17^2$

⇒ $x^2 \pm 16x + 64 + y^2 - 30y + 225 = 289$

⇒ $x^2 + y^2 \pm 16x - 30y = 0$.

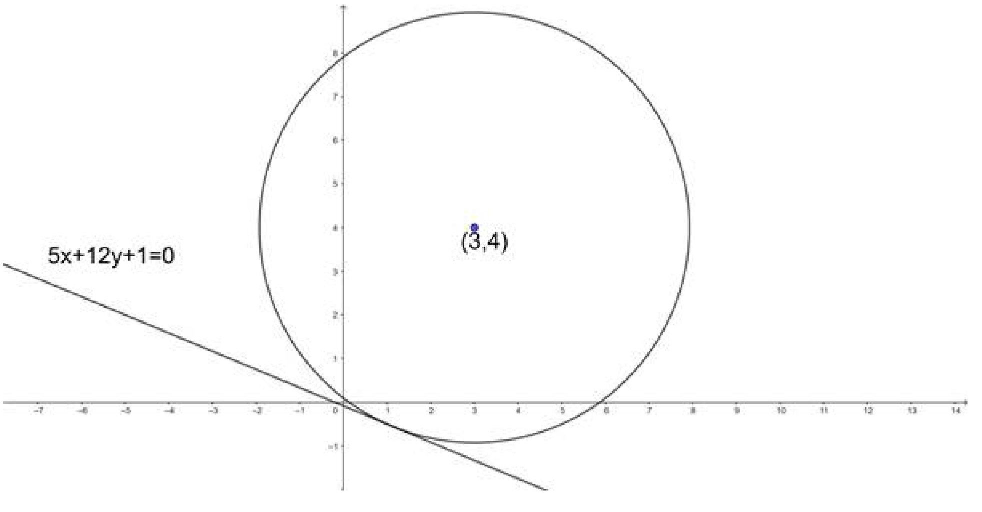
∴The equation of the circle is $x^2 + y^2 \pm 16x - 30y = 0$.

8. Question

Find the equation of the circle which has its centre at the point (3, 4) and touches the straight line 5x + 12y – 1 = 0.

Answer

Given that we need to find the equation of the circle with centre (3, 4) and touches the straight line 5x + 12y - 1 = 0.



Since the circle touches the line at a single point and the circle passes through that point.

We know that the radius of a circle is the distance between the centre and any point that lies on the circle.

Here the point lies on the circle and also on the line, the distance between the points is equal to the perpendicular distance from the centre on to the line 5x + 12y - 1 = 0.

We know that the perpendicular distance from the point (x_1,y_1) on to the line ax + by + c = 0 is given by $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

Let us assume ‘r’ be the radius of the circle.

$$\Rightarrow r = \frac{|5(3) + 12(4) - 1|}{\sqrt{5^2 + 12^2}}$$

$$\Rightarrow r = \frac{|15 + 48 - 1|}{\sqrt{25 + 144}}$$

$$\Rightarrow r = \frac{|62|}{\sqrt{169}}$$

$$\Rightarrow r = \frac{62}{13}.$$

We have a circle with centre (3, 4) and having a radius $\frac{62}{13}$.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x-3)^2 + (y-4)^2 = \left(\frac{62}{13}\right)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 = \frac{3844}{169}$

$\Rightarrow 169x^2 + 169y^2 - 1014x - 1352y + 4225 = 3844$

$\Rightarrow 169x^2 + 169y^2 - 1014x - 1352y + 381 = 0.$

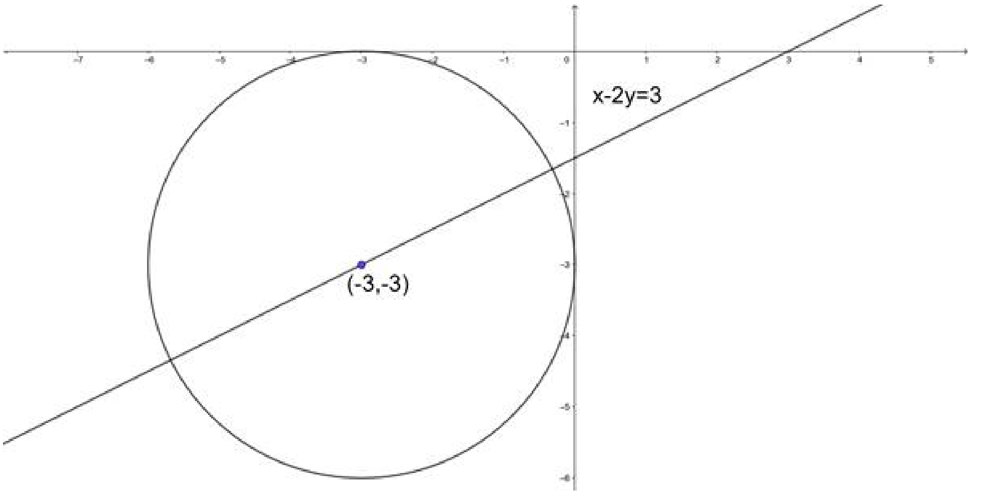
$\therefore$The equation of the circle is $169x^2 + 169y^2 - 1014x - 1352y + 381 = 0$.

9. Question

Find the equation of the circle which touches the axes and whose centre lies on x – 2y = 3.

Answer

We need to find the equation of the circle the axes and centre lies on x - 2y = 3.



Let us assume the circle touches the axes at (a,0) and (0,a) and we get the radius to be |a|.

We get the centre of the circle as (a, a). This point lies on the line x – 2y = 3

$\Rightarrow a - 2(a) = 3$

$\Rightarrow - a = 3$

$\Rightarrow a = - 3$

Centre = (a, a) = (- 3, - 3) and radius of the circle(r) = | - 3| = 3

We have circle with centre (- 3, - 3) and having radius 3.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - (- 3))^2 + (y - (- 3))^2 = 3^2$

$\Rightarrow (x + 3)^2 + (y + 3)^2 = 9$

$\Rightarrow x^2 + 6x + 9 + y^2 + 6y + 9 = 9$

$\Rightarrow x^2 + y^2 + 6x + 6y + 9 = 0.$

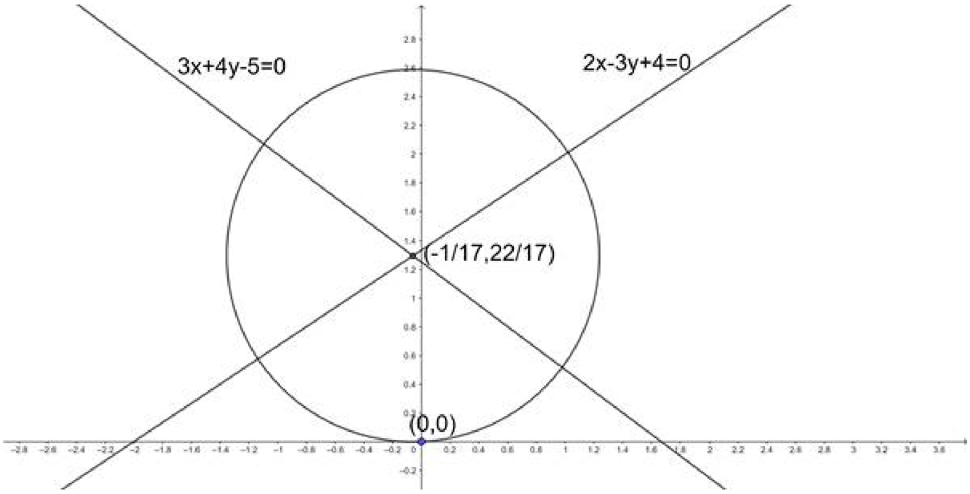
∴The equation of the circle is $x^2 + y^2 + 6x + 6y + 9 = 0$.

10. Question

A circle whose centre is the point of intersection of the lines 2x – 3y + 4 = 0 and 3x + 4y – 5 = 0 passes through the origin. Find its equation.

Answer

Given that the circle has the centre at the intersection point of the lines 2x - 3y + 4 = 0 and 3x + 4y - 5 = 0 and passes through the origin.



Let us find the points of intersection of the lines.

On solving the lines 2x - 3y + 4 = 0 and 3x + 4y - 5 = 0, we get the point of intersection to be $\left(\frac{-1}{17}, \frac{22}{17}\right)$

We have circle with centre $\left(\frac{-1}{17}, \frac{22}{17}\right)$ and passing through the point (0,0).

We know that radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$$\Rightarrow r = \sqrt{\left(\frac{-1}{17} - 0\right)^2 + \left(\frac{22}{17} - 0\right)^2}$$

$$\Rightarrow r = \sqrt{\left(-\frac{1}{17}\right)^2 + \left(\frac{22}{17}\right)^2}$$

$$\Rightarrow r = \sqrt{\frac{1}{289} + \frac{484}{289}}$$

$$\Rightarrow r = \sqrt{\frac{485}{289}}$$

$$\Rightarrow r = \frac{\sqrt{485}}{17} \text{ ..... (1)}$$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$$\Rightarrow \left(x - \left(\frac{-1}{17}\right)\right)^2 + \left(y - \frac{22}{17}\right)^2 = \left(\frac{\sqrt{485}}{17}\right)^2$$

$\Rightarrow \left(x + \frac{1}{17}\right)^2 + \left(y - \frac{22}{17}\right)^2 = \frac{485}{289}$

$\Rightarrow x^2 + \frac{2x}{17} + \frac{1}{289} + y^2 - \frac{44y}{17} + \frac{484}{289} = \frac{485}{289}$

$\Rightarrow 17x^2 + 17y^2 + 2x - 44y = 0$.

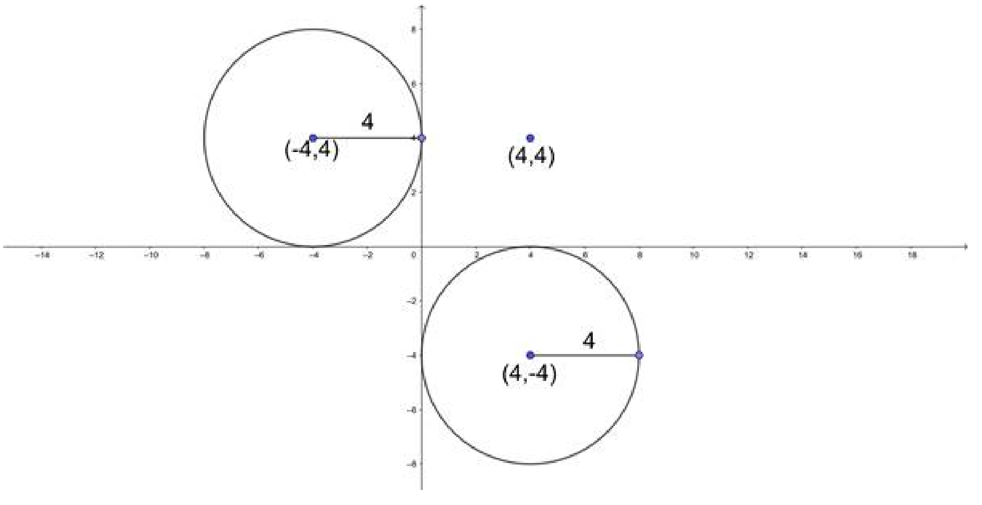
∴The equation of the circle is $17x^2 + 17y^2 + 2x - 44y = 0$.

11. Question

A circle of radius 4 units touches the coordinate axes in the first quadrant. Find the equations of its images with respect to the line mirrors x = 0 and y = 0.

Answer

Given that the circle having radius 4 units touches the coordinate axes in the first quadrant.



Let us assume the circle touches the co - ordinate axes at (a,0) and (0,a). Then the circle will have the centre at (a, a) and radius |a|.

It is given that the radius is 4 units. Since the circle touches the axes in the first quadrant, we will have the centre in the first quadrant.

The centre of the circle is (4, 4).

The centres of the mirrors of the circle w.r.t x = 0 and y = 0 is (- 4, 4) and (4, - 4).

Case (i):

We have a circle with centre (- 4, 4) and having radius 4.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - (-4))^2 + (y - 4)^2 = 4^2$

$\Rightarrow (x + 4)^2 + (y - 4)^2 = 16$

$\Rightarrow x^2 + 8x + 16 + y^2 - 8y + 16 = 16$

$\Rightarrow x^2 + y^2 + 8x - 8y + 16 = 0$.

∴The equation of the circle is $x^2 + y^2 + 8x - 8y + 16 = 0$.

Case (ii):

We have circle with centre (4, - 4) and having radius 4.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x - 4)^2 + (y - (- 4))^2 = 4^2$

⇒ $(x - 4)^2 + (y + 4)^2 = 16$

⇒ $x^2 - 8x + 16 + y^2 + 8y + 16 = 16$

⇒ $x^2 + y^2 - 8x + 8y + 16 = 0.$

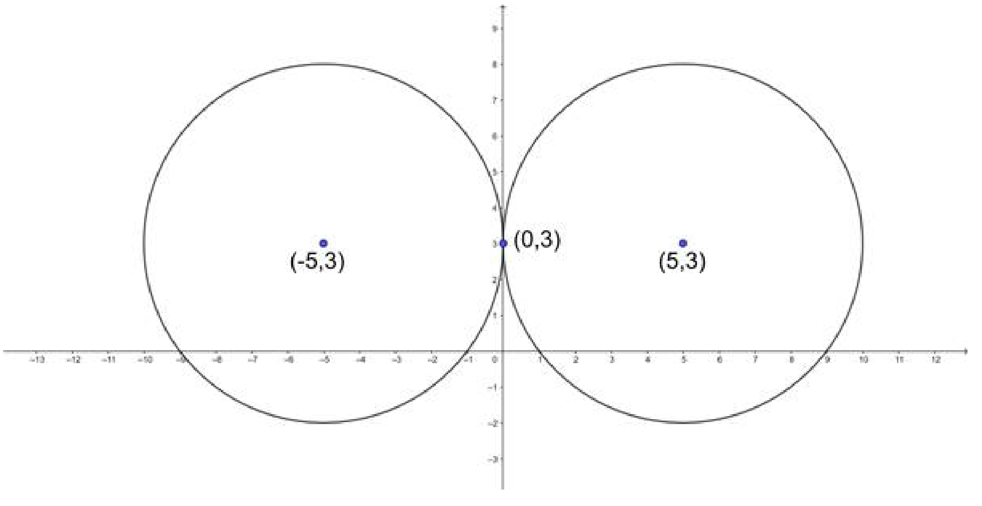
∴The equation of the circle is $x^2 + y^2 - 8x + 8y + 16 = 0$.

12. Question

Find the equations of the circles touching y - axis at (0, 3) and making an intercept of 8 units on the x - axis.

Answer

Given that the circle is touching the y - axis at (0,3) and making an intercept of 8 units on the x - axis.



Let us assume the circle intersects x - axis at the points A, B. Then the length of AB = 8units.

Let us assume ‘O’ be the centre of the circle and ‘M’ be the mid - point of the line AB.

Since circle touches y - axis at (0,3), we assume the centre of the circle be (h,3).

From the figure we get OM = 3 units and AM = 4units. The points AMO forms a right angled triangle. We have AO as radius.

⇒ $AO^2 = OM^2 + AM^2$

⇒ $AO^2 = 3^2 + 4^2$

⇒ $AO^2 = 9 + 16$

⇒ $AO^2 = 25$

⇒ $AO = \sqrt{25}$

⇒ AO = 5

We have circle with centre (h,3), passing through the point (0,3) and having radius 5 units.

We know that radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

⇒ $5 = \sqrt{(h - 0)^2 + (3 - 3)^2}$

$\Rightarrow 5 = \sqrt{(h)^2 + (0)^2}$

$\Rightarrow 5 = |h|$

$\Rightarrow h = \pm 5$ ..... (1)

We have circle with centre (±5,3) and having radius 5 units.

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x \pm 5)^2 + (y - 3)^2 = 5^2$

$\Rightarrow x^2 \pm 10x + 25 + y^2 - 6y + 9 = 25$

$\Rightarrow x^2 + y^2 \pm 10x - 6y + 9 = 0$

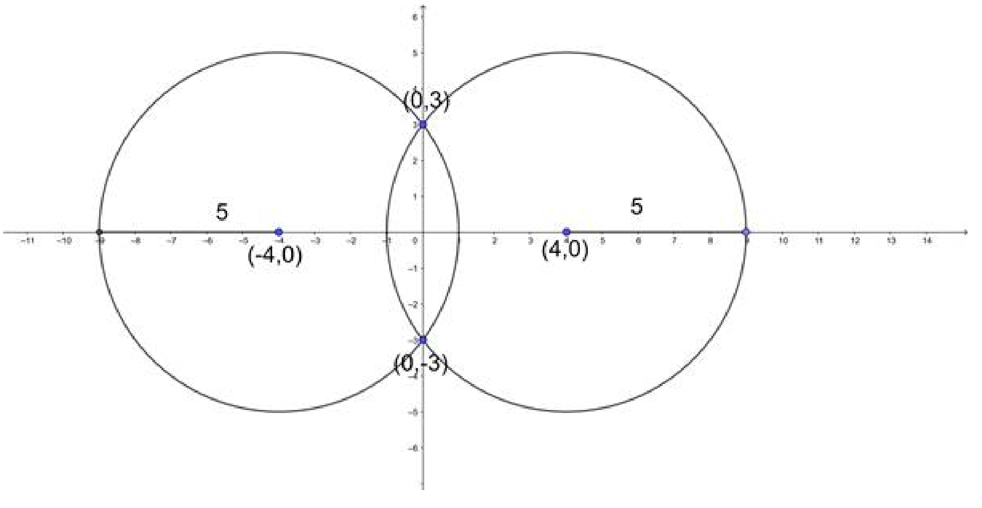
∴The equations of the circles is $x^2 + y^2 \pm 10x - 6y + 9 = 0$.

13. Question

Find the equations of the circles passing through two points on y - axis at distances 3 from the origin and having radius 5.

Answer

Given that we need to find the equations of the circles passing through the points on y - axis at distances 3 from origin and having radius 5.



Since the points are 3 units away from origin on the y - axis, the points will be (0,3) and (0, - 3).

Let us assume the centre of this circle be (h,k).

We know that the equation of the circle with centre (p,q) and having radius 'r' is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - h)^2 + (y - k)^2 = 5^2$

$\Rightarrow (x - h)^2 + (y - k)^2 = 25$ ..... (1)

Since circle passes through the point (0,3). We substitute this point in eq(1)

$\Rightarrow (0 - h)^2 + (3 - k)^2 = 25$

$\Rightarrow h^2 + (3 - k)^2 = 25$ ..... - - (2)

Since circle passes through the point (0, - 3). We substitute this point in eq(1)

$\Rightarrow (0 - h)^2 + (- 3 - k)^2 = 25$

$\Rightarrow h^2 + (3 + k)^2 = 25$ ..... - - (3)

On solving (2) and (3), we get

$\Rightarrow h = \pm 4$ and $k = 0$

We have circle with centre (±4,0) and having radius 5 units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x \pm 4)^2 + (y - 0)^2 = 5^2$

$\Rightarrow x^2 \pm 8x + 16 + y^2 = 25$

$\Rightarrow x^2 + y^2 \pm 8x - 9 = 0$

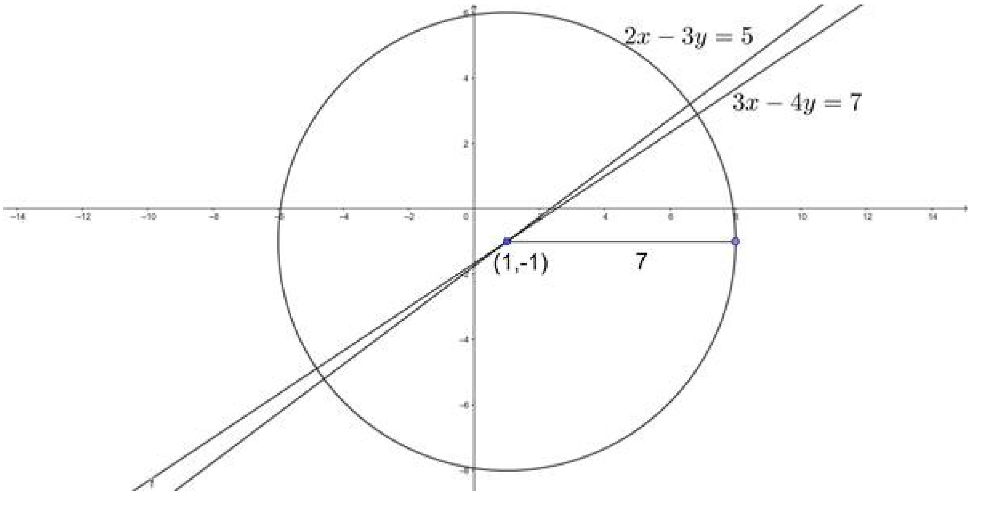
$\therefore$The equations of the circles is $x^2 + y^2 \pm 8x - 9 = 0$.

14. Question

If the lines 2x – 3y = 5 and 3x – 4y = 7 are the diameters of a circle of area 154 squares units, then obtain the equation of the circle.

Answer

Given that we need to find the equation of the circle which has diameters 2x - 3y = 5 and 3x - 4y = 7 and having area 154 square units.



We know that the centre of the circle is the point of intersection of the diameters.

On solving the diameters, we get the centre to be (1, - 1).

We know that the area of the circle is given by πr^2, where r is the radius of the circle.

$\Rightarrow \pi r^2 = 154$

$\Rightarrow r^2 = \frac{154}{\pi}$

$\Rightarrow r = \sqrt{\frac{154}{\pi}}$

$\Rightarrow r = 7$

We have a circle with centre (1, - 1) and having radius 7 units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x - 1)^2 + (y - (-1))^2 = 7^2$

⇒ $(x - 1)^2 + (y + 1)^2 = 49$

⇒ $x^2 - 2x + 1 + y^2 + 2y + 1 = 49$

⇒ $x^2 + y^2 - 2x + 2y - 47 = 0$

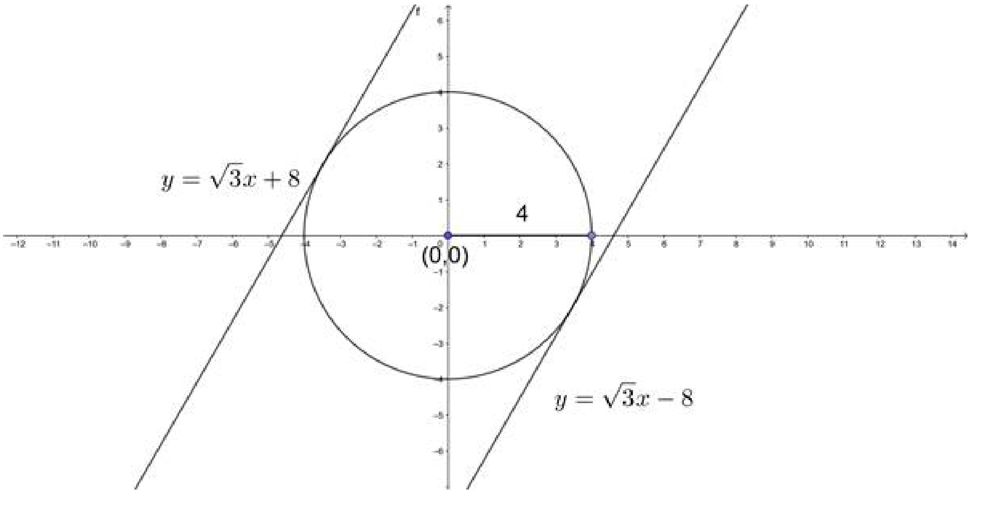
∴The equations of the circles is $x^2 + y^2 - 2x + 2y - 47 = 0$.

15. Question

If the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$, then find the value of k.

Answer

Given that the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$.



Here the circle has centre at (0, 0) and radius 4.

The line touches the circle at a point. So, the distance between this point and the centre is equal to the radius of the circle.

This distance is the same as the perpendicular distance between the centre and the line.

We know that the perpendicular distance from the point (x_1,y_1) to the line ax + by + c = 0 is given by $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

$$\Rightarrow 4 = \frac{|\sqrt{3}(0) - 0 + k|}{\sqrt{(\sqrt{3})^2 + 1^2}}$$

$$\Rightarrow 4 = \frac{|k|}{\sqrt{3 + 1}}$$

$$\Rightarrow 4 = \frac{|k|}{2}$$

⇒ $|k| = 8$

⇒ $k = \pm 8$

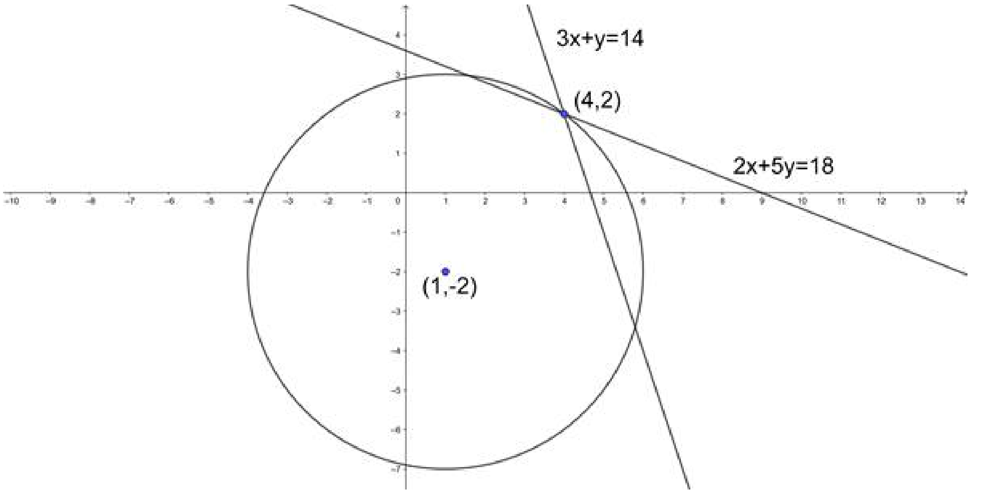
∴The value of k is ±8.

16. Question

Find the equation of the circle having (1, - 2) as its centre and passing through the intersection of the lines 3x + y = 14 and 2x + 5y = 18.

Answer

Given that we need to find the equation of the circle with centre (1, - 2) and passing through the point of intersection of the lines 3x + y = 14 and 2x + 5y = 18.



Let us first find the point of intersection of lines 3x + y = 14 and 2x + 5y = 18.

On solving the lines, we get the intersection point to be (4,2).

We have a circle with centre (1, - 2) and passing through the point (4,2).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(1-4)^2 + (-2-2)^2}$

$\Rightarrow r = \sqrt{(-3)^2 + (-4)^2}$

$\Rightarrow r = \sqrt{9 + 16}$

$\Rightarrow r = \sqrt{25}$

$\Rightarrow r = = 5$ ..... (1)

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - 1)^2 + (y - (- 2))^2 = (5)^2$

$\Rightarrow (x - 1)^2 + (y + 2)^2 = 25$

$\Rightarrow x^2 - 2x + 1 + y^2 + 4y + 4 = 25$

$\Rightarrow x^2 + y^2 - 2x + 4y - 20 = 0.$

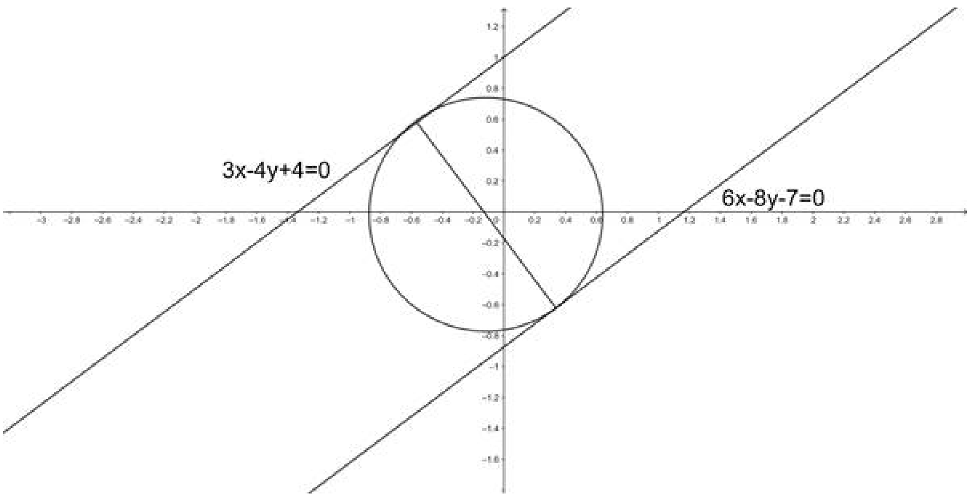
∴The equation of the circle is $x^2 + y^2 - 2x + 4y - 20 = 0$.

17. Question

If the lines 3x – 4y + 4 = 0 and 6x – 8y – 7 = 0 are tangents to a circle, then find the radius of the circle.

Answer

Given that the lines are 3x - 4y + 4 = 0 and 6x - 8y - 7 = 0 are the tangents to the circle.



Let us find the slope of each line.

We know that the slope of the line ax + by + c = 0 is $\frac{-b}{a}$.

⇒ The slope of the line 3x - 4y + 4 = 0 is $\frac{-(-4)}{3} = \frac{4}{3}$

⇒ The slope of the line 6x - 8y - 7 = 0 is $\frac{-(-8)}{6} = \frac{4}{3}$

The slopes are equal. So, these are the tangents on either side of the diameter of the circle as shown in the figure.

The length of the diameter is the perpendicular distance between these two parallel lines.

We know that the distance between the two parallel lines ax + by + c_1 = 0 and ax + by + c_2 = 0 is given by $\frac{|C_1 - C_2|}{\sqrt{a^2 + b^2}}$

Here the parallel are 3x - 4y + 4 = 0 and 6x - 8y - 7 = 0(or $3x - 4y - \frac{7}{2} = 0$)

Let ‘d’ and ‘r’ be the diameter and radius of the circle.

$\Rightarrow d = \frac{\left|4-\left(\frac{-7}{2}\right)\right|}{\sqrt{3^2 + 4^2}}$

$\Rightarrow 2r = \frac{\left|4 + \frac{7}{2}\right|}{\sqrt{9 + 16}}$

$\Rightarrow 2r = \frac{\left|\frac{15}{2}\right|}{\sqrt{25}}$

$\Rightarrow 2r = \frac{\left|\frac{15}{2}\right|}{5}$

$\Rightarrow 2r = \frac{3}{2}$

$\Rightarrow r = \frac{3}{4}$

∴The radius of the circle is $\frac{3}{4}$ units.

18. Question

Show that the point (x, y) given by $x = \frac{2at}{1+t^2}$ and $y = a\left(\frac{1-t^2}{1+t^2}\right)$ lies on a circle for all real values of t such that - 1 ≤ t ≤ 1, where a is any given real number.

Answer

Given:

$\Rightarrow x = \frac{2at}{1+t^2}$

$\Rightarrow y = a\left(\frac{1-t^2}{1+t^2}\right)$

We need to prove that the point (x, y) lies on a circle for real values of t such that - 1≤t≤1, where a is any given real number.

Consider $x^2 + y^2$,

$$\Rightarrow x^2 + y^2 = \left(\frac{2at}{1+t^2}\right)^2 + \left(a\left(\frac{1-t^2}{1+t^2}\right)\right)^2$$

$$\Rightarrow x^2 + y^2 = \left(\frac{4a^2t^2}{(1+t^2)}\right) + \left(a^2\left(\frac{1-2t^2+t^4}{(1+t^2)^2}\right)\right)$$

$$\Rightarrow x^2 + y^2 = \frac{4a^2t^2 + a^2 - 2a^2t^2 + a^2t^4}{(1+t^2)^2}$$

$$\Rightarrow x^2 + y^2 = \frac{a^2 + 2a^2t^2 + a^2t^4}{(1+t^2)^2}$$

$$\Rightarrow x^2 + y^2 = \frac{a^2(1+2t^2+t^4)}{(1+t^2)^2}$$

$$\Rightarrow x^2 + y^2 = \frac{a^2(1+t^2)^2}{(1+t^2)^2}$$

$\Rightarrow x^2 + y^2 = a^2$

The point (x,y) lies on a circle.

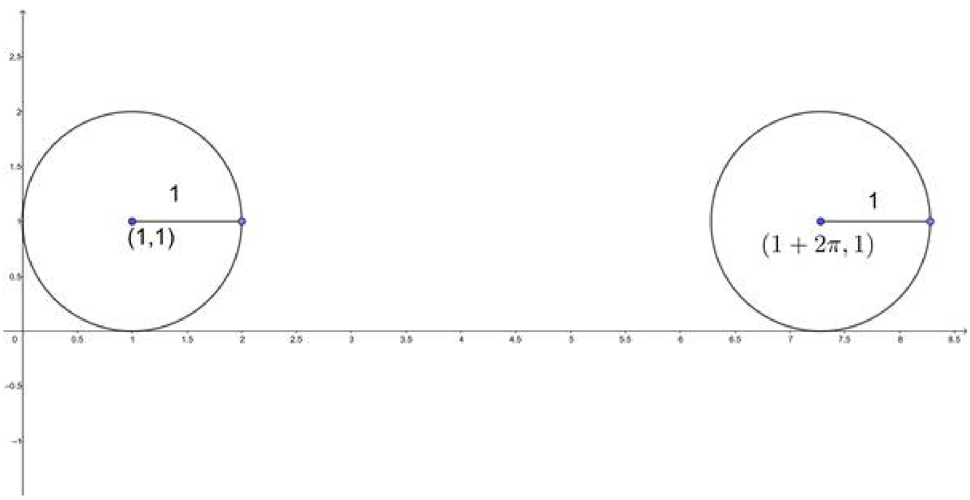
∴Thus proved.

19. Question

The circle $x^2 + y^2 - 2x - 2y + 1 = 0$ is rolled along the positive direction of x - axis and makes one complete roll. Find its equation in new - position.

Answer

Given equation of the circle is $x^2 + y^2 - 2x - 2y + 1 = 0$.



We know that the equation of the circle with centre (p,q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$ ..... - (1)

Now,

$\Rightarrow x^2 + y^2 - 2x - 2y + 1 = 0$

⇒ $(x^2 - 2x + 1) + (y^2 - 2y + 1) = 1$

⇒ $(x - 1)^2 + (y - 1)^2 = (1)^2$ ..... (2)

Comparing (2) with (1), we get

⇒ Centre = (1,1) and radius = 1

It is told that the circle is rolled along the positive direction of x - axis and makes one complete roll.

We know that the complete roll of a circle covers the distance $2\pi r$, where r is the radius of the circle.

The centre of the circle as moves $2\pi r$ in the positive direction of xthe - axis.

Let the d be the distance moved by the centre on completion of one roll.

⇒ $d = 2\pi(1)$

⇒ $d = 2\pi$

The new position of the centre is (1 + d,1)

⇒ Centre = $(1 + 2\pi, 1)$

We have circle with centre $(1 + 2\pi, 1)$ and having radius 1 units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x - (1 + 2\pi))^2 + (y - 1)^2 = 1^2$

⇒ $(x - 1 - 2\pi)^2 + (y - 1)^2 = 1$

⇒ $x^2 - (2 + 4\pi)x + (1 + 2\pi)^2 + y^2 - 2y + 1 = 1$

⇒ $x^2 + y^2 - (2 + 4\pi)x - 2y - (1 + 2\pi)^2 = 0$

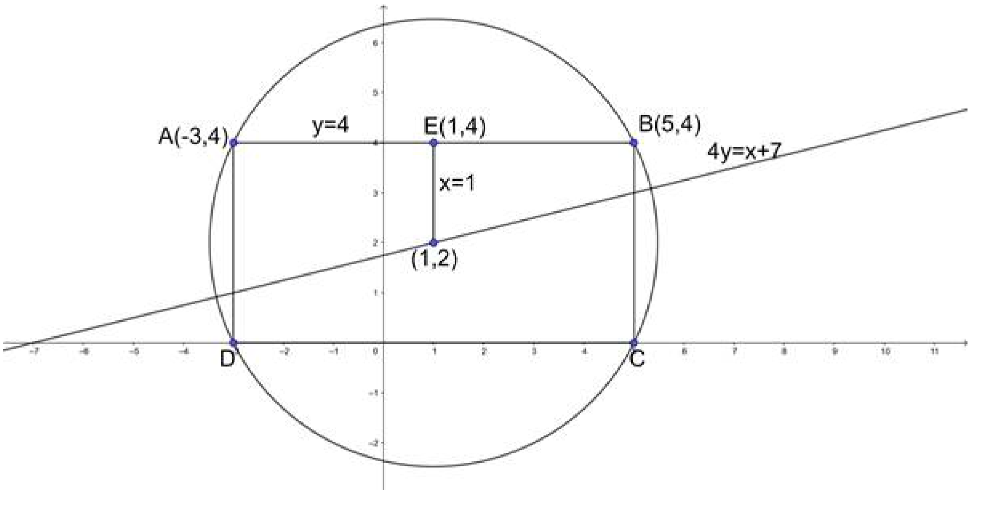
∴The equations of the circles is $x^2 + y^2 - (2 + 4\pi)x - 2y - (1 + 2\pi)^2 = 0$.

20. Question

One diameter of the circle circumscribing the rectangle ABCD is 4y = x + 7. If the coordinates of A and B are (- 3, 4) and (5, 4) respectively, find the equation of the circle.

Answer

Given that x - 4y + 7 = 0 is one of the diameters of the circle circumscribing the rectangle ABCD.



The coordinates of A and B are (- 3,4) and (5,4).

Let us assume E be the mid - point of the line AB and ‘O’ be the centre of the circle.

$\Rightarrow E = \left(\frac{-3+5}{2}, \frac{4+4}{2}\right)$

$\Rightarrow E = \left(\frac{2}{2}, \frac{8}{2}\right)$

$\Rightarrow E = (1,4)$ - - - - - - (1)

Let us find the equation of the line AB.

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - 4 = \frac{4-4}{5-(-3)}(x - (-3))$

$\Rightarrow y - 4 = 0(x + 3)$

⇒ y = 4 ..... - (2)

From the figure, we can see that the line EO is perpendicular to the line AB.

So, the equation of the line EO is x = 1.

We find the centre as it is the point of intersection of the lines x - 4y + 7 = 0 and x = 1.

On solving this, we get the centre to be (1, 2).

We have a circle with centre (1, 2) and passing through the point (5,4).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(1-4)^2 + (2-4)^2}$

$\Rightarrow r = \sqrt{(-3)^2 + (-2)^2}$

$\Rightarrow r = \sqrt{9 + 4}$

$\Rightarrow r = \sqrt{13}$ ..... (2)

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x-1)^2 + (y-2)^2 = \left(\sqrt{13}\right)^2$

⇒ $x^2 - 2x + 1 + y^2 - 4y + 4 = 13$

⇒ $x^2 + y^2 - 2x - 4y - 8 = 0$.

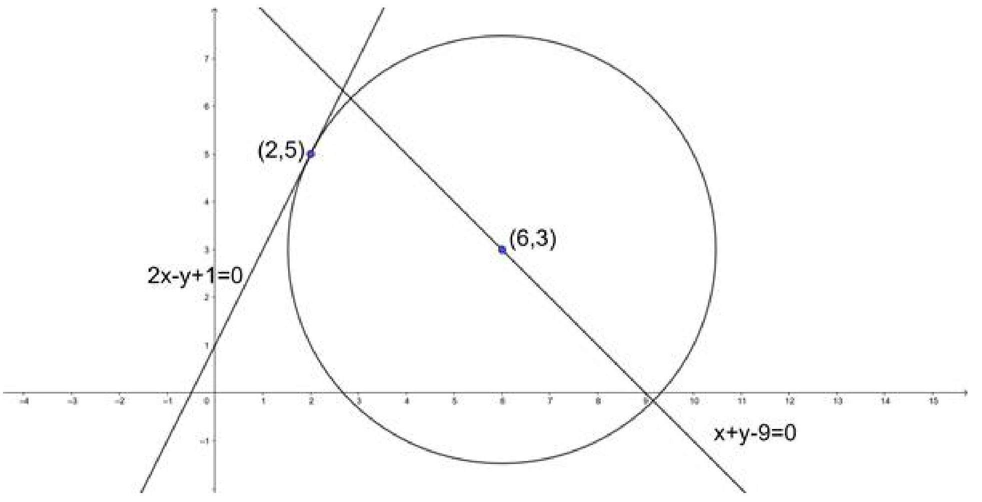
∴The equation of the circle is $x^2 + y^2 - 2x - 4y - 8 = 0$.

21. Question

If the line 2x – y + 1 = 0 touches the circle at the point (2, 5) and the centre of the circle lies on the line x + y – 9 = 0. Find the equation of the circle.

Answer

Given that the line 2x - y + 1 = 0 touches the circle at the point (2,5) and the centre lies on the line x + y - 9 = 0.



We know that normal at any point on the circle passes through the centre of the circle.

We have tangent 2x - y + 1 = 0 at the point (2,5) for the circle.

We know that normal is perpendicular to the tangent and passes through the point of contact.

We know that product of slopes of two perpendicular lines is - 1.

Let us assume m be the slope of the normal.

We know that slope of a line ax + by + c = 0 is $\frac{-a}{b}$.

Slope of the line 2x - y + 1 = 0 is $\frac{-2}{-1} = 2$

⇒ m.(2) = - 1

⇒ $m = \frac{-1}{2}$ - - - - - (1)

We know that the equation of a straight line passing through the point (x_1, y_1) and having slope ‘m’ is given by (y - y_1) = m(x - x_1).

⇒ $y - 5 = (\frac{-1}{2})(x - 2)$

⇒ 2y - 10 = - x + 2

⇒ x + 2y - 12 = 0 ..... (2)

We get the centre by solving for the intersecting point of the lines x + 2y - 12 = 0 and x + y - 9 = 0 as they both pass through the centre.

On solving these two lines we get,

⇒ Centre = (6, 3)

We have circle with centre (6, 3) and passing through the point (2, 5).

We know that radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

⇒ $r = \sqrt{(6-2)^2 + (3-5)^2}$

⇒ $r = \sqrt{(4)^2 + (-2)^2}$

⇒ $r = \sqrt{16 + 4}$

⇒ $r = \sqrt{20}$ ..... (2)

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x-6)^2 + (y-3)^2 = (\sqrt{20})2$

⇒ $x^2 - 12x + 36 + y^2 - 6y + 9 = 20$

⇒ $x^2 + y^2 - 12x - 6y + 25 = 0$.

∴The equation of the circle is $x^2 + y^2 - 12x - 6y + 25 = 0$.

Exercise 24.2

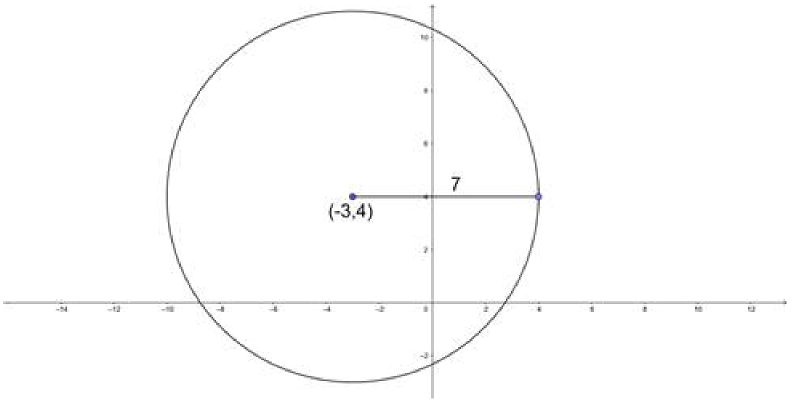
1 A. Question

Find the coordinates of the centre radius of each of the following circle:

$x^2 + y^2 + 6x - 8y - 24 = 0$

Answer

Given equation of the circle is $x^2 + y^2 + 6x - 8y - 24 = 0$ …… (1)



We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$ ……(2)

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

Comparing (1) with (2) we get,

⇒ Centre = $\left(\frac{-6}{2}, \frac{-(-8)}{2}\right)$

⇒ Centre = (- 3,4)

⇒ Radius = $\sqrt{3^2 + 4^2 - (-24)}$

⇒ Radius = $\sqrt{9 + 16 + 24}$

⇒ Radius = √49

⇒ Radius = 7

∴ The centre and radius of the circle is (- 3,4) and 7.

1 B. Question

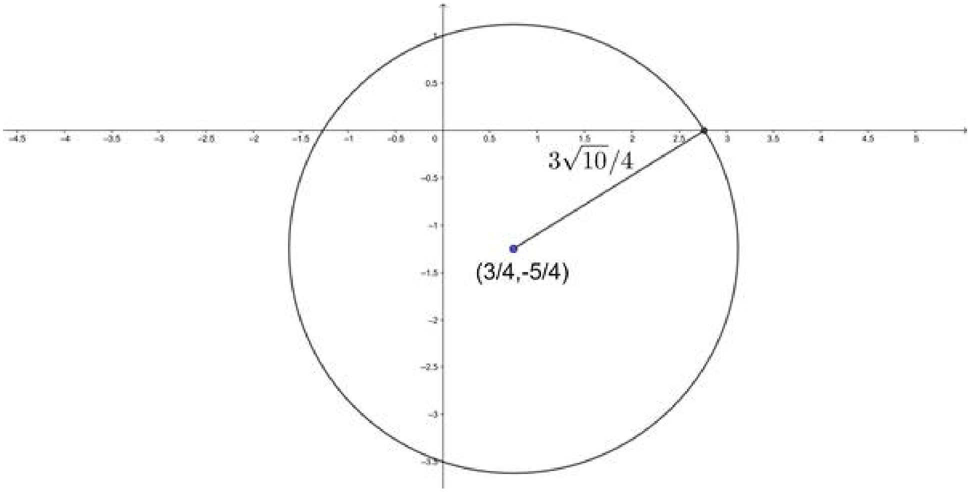
Find the coordinates of the centre radius of each of the following circle:

$2x^2 + 2y^2 – 3x + 5y = 7$

Answer

Given equation of the circle is $2x^2 + 2y^2 - 3x + 5y = 7$

$\Rightarrow x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0$ ..... (1)



We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$ ...... (2)

$\Rightarrow$ Centre = (- a, - b)

$\Rightarrow$ Radius = $\sqrt{a^2 + b^2 - c}$

Comparing (1) with (2) we get,

$\Rightarrow$ Centre = $\left(\frac{-\left(\frac{-3}{2}\right)}{2}, \frac{-\left(\frac{5}{2}\right)}{2}\right)$

$\Rightarrow$ Centre = $\left(\frac{3}{4}, \frac{-5}{4}\right)$

$\Rightarrow$ Radius = $\sqrt{\left(\frac{3}{4}\right)^2 + \left(-\frac{5}{4}\right)^2 - \left(\frac{-7}{2}\right)}$

$\Rightarrow$ Radius = $\sqrt{\frac{9}{16} + \frac{25}{16} + \frac{7}{2}}$

$\Rightarrow$ Radius = $\sqrt{\frac{90}{16}}$

$\Rightarrow$ Radius = $\frac{3\sqrt{10}}{4}$

$\therefore$ The centre and radius of the circle is $\left(\frac{3}{4}, \frac{-5}{4}\right)$ and $\frac{3\sqrt{10}}{4}$.

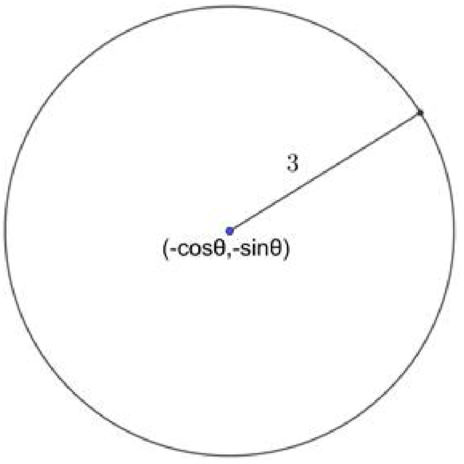
1 C. Question

Find the coordinates of the centre radius of each of the following circle:

$$\frac{1}{2}(x^2 + y^2) + x\cos\theta + y\sin\theta - 4 = 0$$

Answer

Given: the equation of the circle is $\frac{1}{2}(x^2 + y^2) + x\cos\theta + y\sin\theta - 4 = 0$



⇒ $x^2 + y^2 + 2x\cos\theta + 2y\sin\theta - 8 = 0$ ..... (1)

We know that for a circle

$x^2 + y^2 + 2ax + 2by + c = 0$ .... (2)

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

Comparing (1) with (2) we get,

⇒ Centre = $\left(\frac{-2\cos\theta}{2}, \frac{-2\sin\theta}{2}\right)$

⇒ Centre = (- cosθ, - sinθ)

⇒ Radius = $\sqrt{(-\cos\theta)^2 + (\sin\theta)^2 - (-8)}$

⇒ Radius = $\sqrt{\cos^2\theta + \sin^2\theta + 8}$

⇒ Radius = $\sqrt{1 + 8}$

⇒ Radius = √9

⇒ Radius = 3

∴ The centre and radius of the circle is (- cosθ, - sinθ) and 3.

1 D. Question

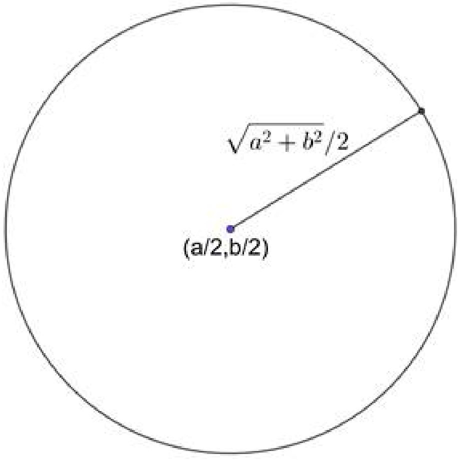
Find the coordinates of the centre radius of each of the following circle:

$x^2 + y^2 - ax - by = 0$

Answer

Given:

equation of the circle is $x^2 + y^2 - ax - by = 0$ .... (1)



We know that for a circle

$x^2 + y^2 + 2ax + 2by + c = 0$ .... (2)

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

Comparing (1) with (2) we get,

⇒ Centre = $\left(\frac{-(-a)}{2}, \frac{-(-b)}{2}\right)$

⇒ Centre = $\left(\frac{a}{2}, \frac{b}{2}\right)$

⇒ Radius = $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

⇒ Radius = $\sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$

⇒ Radius = $\sqrt{\frac{a^2 + b^2}{4}}$

⇒ Radius = $\frac{\sqrt{a^2 + b^2}}{2}$

∴ The centre and radius of the circle is $\left(\frac{a}{2}, \frac{b}{2}\right)$ and $\frac{\sqrt{a^2 + b^2}}{2}$.

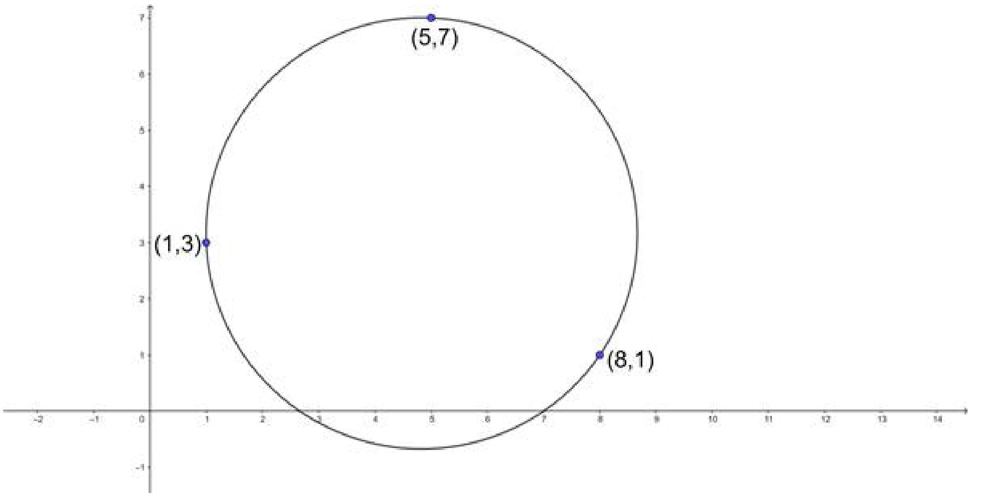
2 A. Question

Find the equation of the circle passing through the points :

(5, 7), (8, 1) and (1, 3)

Answer

Given that we need to find the equation of the circle passing through the points (5,7), (8,1) and (1,3).



We know that the standard form of the equation of a circle is given by:

$\Rightarrow x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)

Substituting (5,7) in (1), we get

$\Rightarrow 5^2 + 7^2 + 2a(5) + 2b(7) + c = 0$

$\Rightarrow 25 + 49 + 10a + 14b + c = 0$

$\Rightarrow 10a + 14b + c + 74 = 0$ ..... (2)

Substituting (8,1) in (1), we get

$\Rightarrow 8^2 + 1^2 + 2a(8) + 2b(1) + c = 0$

$\Rightarrow 64 + 1 + 16a + 2b + c = 0$

$\Rightarrow 16a + 2b + c + 65 = 0$ ..... (3)

Substituting (1,3) in (1), we get

$\Rightarrow 1^2 + 3^2 + 2a(1) + 2b(3) + c = 0$

$\Rightarrow 1 + 9 + 2a + 6b + c = 0$

$\Rightarrow 2a + 6b + c + 10 = 0$ ..... (4)

Solving (2), (3), (4) we get

$\Rightarrow a = \frac{-29}{6}, b = \frac{-19}{6}, c = \frac{56}{3}.$

Substituting these values in (1), we get

$\Rightarrow x^2 + y^2 + 2\left(\frac{-29}{6}\right)x + 2\left(\frac{-19}{6}\right) + \frac{56}{3} = 0$

$\Rightarrow x^2 + y^2 - \frac{29}{3}x - \frac{19}{3}y + \frac{56}{3} = 0$

$\Rightarrow 3x^2 + 3y^2 - 29x - 19y + 56 = 0$

$\therefore$ The equation of the circle is $3x^2 + 3y^2 - 29x - 19y + 56 = 0$.

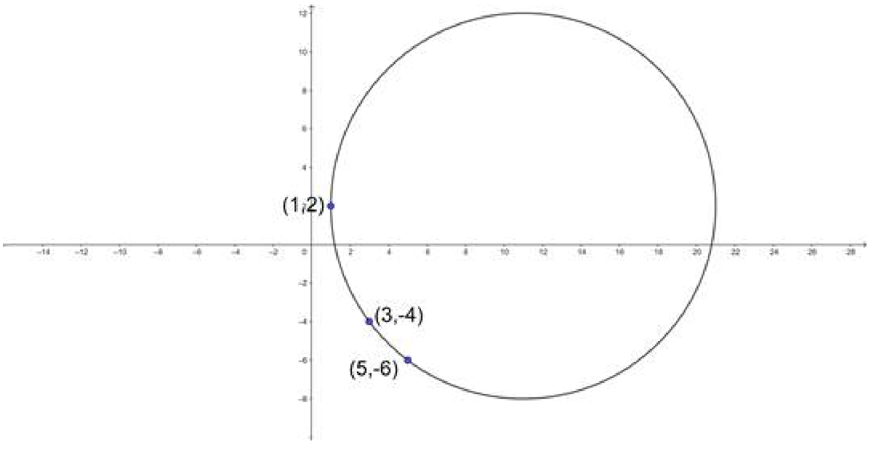
2 B. Question

Find the equation of the circle passing through the points :

(1, 2), (3, - 4) and (5, - 6)

Answer

Given that we need to find the equation of the circle passing through the points (1,2), (3, - 4) and (5, - 6).



We know that the standard form of the equation of a circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)

Substituting (1,2) in (1), we get

⇒ $1^2 + 2^2 + 2a(1) + 2b(2) + c = 0$

⇒ $1 + 4 + 2a + 4b + c = 0$

⇒ $2a + 4b + c + 5 = 0$ ..... (2)

Substituting (3, - 4) in (1), we get

⇒ $3^2 + (-4)^2 + 2a(3) + 2b(-4) + c = 0$

⇒ $9 + 16 + 6a - 8b + c = 0$

⇒ $6a - 8b + c + 25 = 0$ ..... (3)

Substituting (5, - 6) in (1), we get

⇒ $5^2 + (-6)^2 + 2a(5) + 2b(-6) + c = 0$

⇒ $25 + 36 + 10a - 12b + c = 0$

⇒ $10a - 12b + c + 61 = 0$ .....(4)

Solving (2), (3), (4) we get

⇒ $a = -11, b = -2, c = 25$.

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2(-11)x + 2(-2) + 25 = 0$

⇒ $x^2 + y^2 - 22x - 4y + 25 = 0$

∴ The equation of the circle is $x^2 + y^2 - 22x - 4y + 25 = 0$.

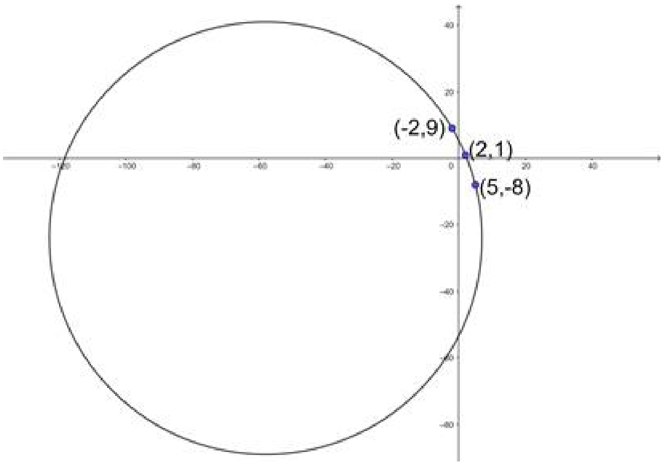
2 C. Question

Find the equation of the circle passing through the points :

(5, - 8), (- 2, 9) and (2, 1)

Answer

Given that we need to find the equation of the circle passing through the points (5, - 8), (- 2,9) and (2,1).



We know that the standard form of the equation of a circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)

Substituting (5, - 8) in (1), we get

⇒ $5^2 + (-8)^2 + 2a(5) + 2b(-8) + c = 0$

⇒ $25 + 64 + 10a - 16b + c = 0$

⇒ $10a - 16b + c + 89 = 0$ ..... (2)

Substituting (- 2,9) in (1), we get

⇒ $(-2)^2 + 9^2 + 2a(-2) + 2b(9) + c = 0$

⇒ $4 + 81 - 4a + 18b + c = 0$

⇒ $-4a + 18b + c + 85 = 0$ ..... (3)

Substituting (2,1) in (1), we get

⇒ $2^2 + 1^2 + 2a(2) + 2b(1) + c = 0$

⇒ $4 + 1 + 4a + 2b + c = 0$

⇒ $4a + 2b + c + 5 = 0$ ..... (4)

Solving (2), (3), (4) we get

⇒ $a = 58$, $b = 24$, $c = -285$.

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2(58)x + 2(24) - 285 = 0$

⇒ $x^2 + y^2 + 116x + 48y - 285 = 0$

∴ The equation of the circle is $x^2 + y^2 + 116x + 48y - 285 = 0$.

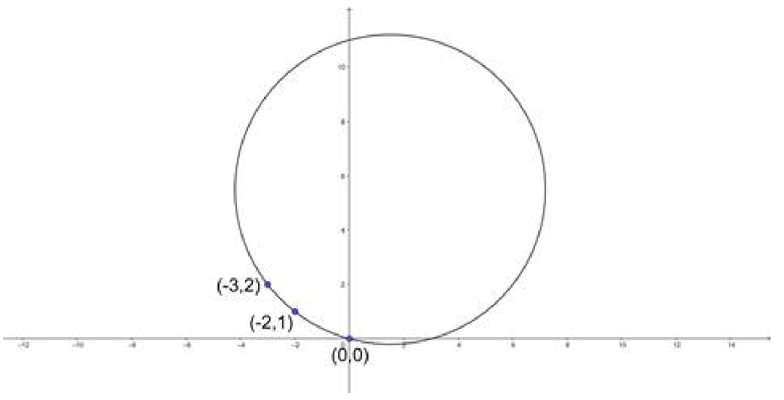
2 D. Question

Find the equation of the circle passing through the points :

(0, 0), (- 2, 1) and (- 3, 2)

Answer

Given that we need to find the equation of the circle passing through the points (0,0), (- 2,1) and (- 3, 2).



We know that the standard form of the equation of a circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)

Substituting (0,0) in (1), we get

⇒ $0^2 + 0^2 + 2a(0) + 2b(0) + c = 0$

⇒ $0 + 0 + 0a + 0b + c = 0$

⇒ $c = 0$ ..... (2)

Substituting (- 2,1) in (1), we get

⇒ $(-2)^2 + 1^2 + 2a(-2) + 2b(1) + c = 0$

⇒ $4 + 1 - 4a + 2b + c = 0$

⇒ $-4a + 2b + c + 5 = 0$ ..... (3)

Substituting (- 3,2) in (1), we get

⇒ $(-3)^2 + 2^2 + 2a(-3) + 2b(2) + c = 0$

⇒ $9 + 4 - 6a + 4b + c = 0$

⇒ $-6a + 4b + c + 13 = 0$ ..... (4)

Solving (2), (3), (4) we get

⇒ $a = \frac{-3}{2}, b = \frac{-11}{2}, c = 0.$

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2\left(\frac{-3}{2}\right)x + 2\left(\frac{-11}{2}\right)y + 0 = 0$

⇒ $x^2 + y^2 - 3x - 11y = 0$

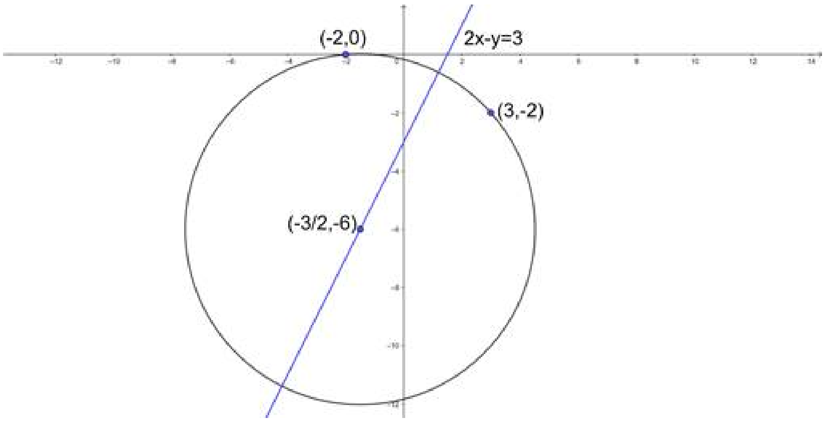
∴ The equation of the circle is $x^2 + y^2 - 3x - 11y = 0$.

3. Question

Find the equation of the circle which passes through (3, - 2), (- 2, 0) and has its centre on the line 2x – y = 3.

Answer

Given that we need to find the equation of the circle which passes through (3, - 2), (- 2,0) and has its centre on the line 2x - y = 3. ..... (1)



We know that the standard form of the equation of the circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ .....(2)

Substituting centre (- a, - b) in (1) we get,

⇒ 2(- a) - (- b) = 3

⇒ - 2a + b = 3

⇒ 2a - b + 3 = 0 ......(3)

Substituting (3, - 2) in (2), we get

⇒ $3^2 + (-2)^2 + 2a(3) + 2b(-2) + c = 0$

⇒ 9 + 4 + 6a - 4b + c = 0

⇒ 6a - 4b + c + 13 = 0 ..... (4)

Substituting (- 2,0) in (2), we get

⇒ $(-2)^2 + 0^2 + 2a(-2) + 2b(0) + c = 0$

⇒ 4 + 0 - 4a + c = 0

⇒ 4a - c - 4 = 0 ..... (5)

Solving (3), (4) and (5) we get,

⇒ $a = \frac{3}{2}, b = 6, c = 2$

Substituting these values in (2), we get

⇒ $x^2 + y^2 + 2\left(\frac{3}{2}\right)x + 2(6)y + 2 = 0$

⇒ $x^2 + y^2 + 3x + 12y + 2 = 0$

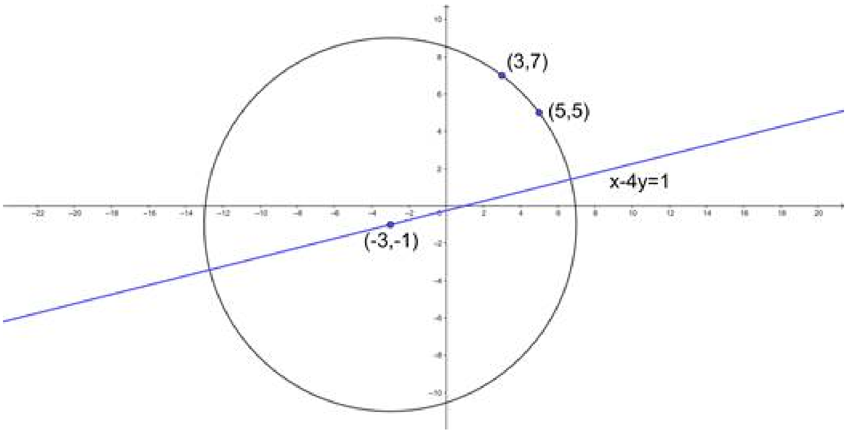
∴ The equation of the circle is $x^2 + y^2 + 3x + 12y + 2 = 0$.

4. Question

Find the equation of the circle which passes through the points (3, 7), (5, 5) and has its centre on line x – 4y = 1.

Answer

Given that we need to find the equation of the circle which passes through (3,7), (5,5) and has its centre on the line x - 4y = 1. ..... (1)



We know that the standard form of the equation of the circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ .....(2)

Substituting centre (- a, - b) in (1) we get,

⇒ $(-a) - 4(-b) = 1$

⇒ $-a + 4b = 1$

⇒ $a - 4b + 1 = 0$ ......(3)

Substituting (3,7) in (2), we get

⇒ $3^2 + 7^2 + 2a(3) + 2b(7) + c = 0$

⇒ $9 + 49 + 6a + 14b + c = 0$

⇒ $6a + 14b + c + 58 = 0$ ..... (4)

Substituting (5,5) in (2), we get

⇒ $5^2 + 5^2 + 2a(5) + 2b(5) + c = 0$

⇒ $25 + 25 + 10a + 10b + c = 0$

⇒ $10a + 10b + c + 50 = 0$ ..... (5)

Solving (3), (4) and (5) we get,

⇒ $a = 3, b = 1, c = -90$

Substituting these values in (2), we get

⇒ $x^2 + y^2 + 2(3)x + 2(1)y - 90 = 0$

⇒ $x^2 + y^2 + 6x + 2y - 90 = 0$

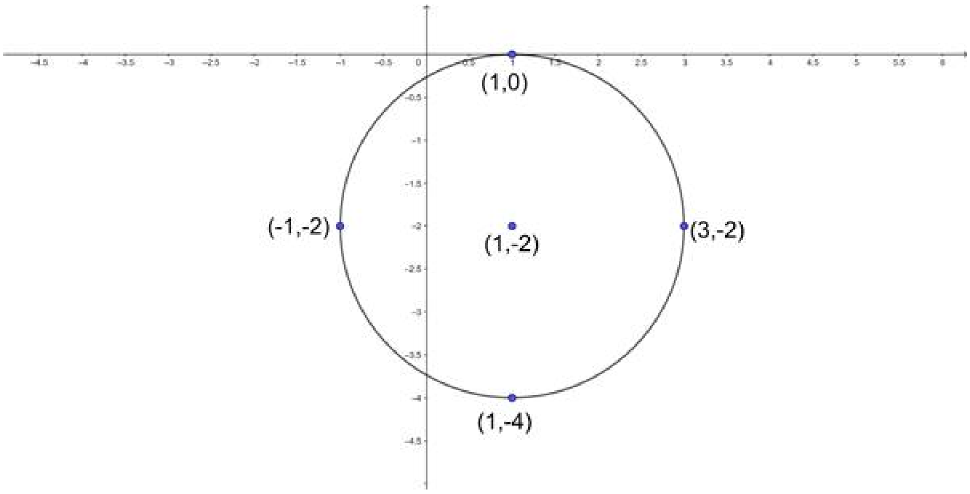
∴ The equation of the circle is $x^2 + y^2 + 6x + 2y - 90 = 0$.

5. Question

Show that the points (3, - 2), (1, 0), (- 1, - 2) and (1, - 4) are con - cyclic.

Answer

Given that we need to show the points A(3, - 2), B(1,0), C(- 1, - 2) and D(1, - 4) are con - cyclic.



The term con - cyclic means the points lie on the same circle.

Let us make a circle with any three points and check whether the fourth point lies on it or not.

Let us assume the circle passes through the points A, B, C.

We know that the standard form of the equation of the circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)

Substituting A(3, - 2) in (1), we get,

⇒ $3^2 + (- 2)^2 + 2a(3) + 2b(- 2) + c = 0$

⇒ $9 + 4 + 6a - 4b + c = 0$

⇒ $6a - 4b + c + 13 = 0$ ..... (2)

Substituting B(1,0) in (1), we get,

⇒ $1^2 + 0^2 + 2a(1) + 2b(0) + c = 0$

⇒ $1 + 2a + c = 0$ ......- (3)

Substituting C(- 1, - 2) in (1), we get,

⇒ $(- 1)^2 + (- 2)^2 + 2a(- 1) + 2b(- 2) + c = 0$

⇒ $1 + 4 - 2a - 4b + c = 0$

⇒ $5 - 2a - 4b + c = 0$

⇒ $2a + 4b - c - 5 = 0$ ..... (4)

On solving (2), (3) and (4) we get,

⇒ a = - 1, b = 2 and c = 1

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2(- 1)x + 2(2)y + 1 = 0$

⇒ $x^2 + y^2 - 2x + 4y + 1 = 0$ ..... (5)

Substituting D(1, - 4) in eq(5) we get,

⇒ $1^2 + (- 4)^2 - 2(1) + 4(- 4) + 1$

⇒ $1 + 16 - 2 - 16 + 1$

⇒ 0

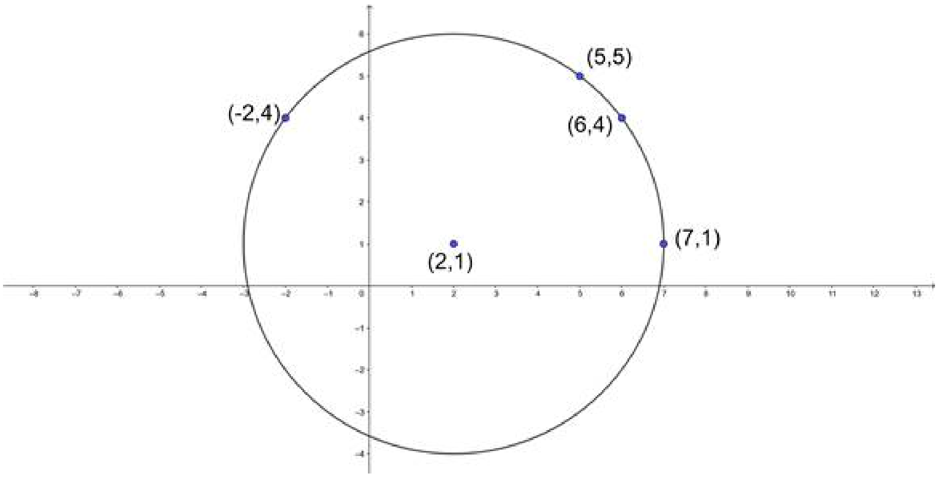
∴ The points (3, - 2), (1,0), (- 1, - 2), (1, - 4) are con - cyclic.

6. Question

Show that the points (5, 5), (6, 4), (- 2, 4) and (7, 1) all lie on a circle, and find its equation, centre, and radius.

Answer

Given that we need to show the points A(5,5), B(6,4), C(- 2,4) and D(7,1) lie on a circle.



Let us make a circle with any three points and check whether the fourth point lies on it or not.

Let us assume the circle passes through the points A, B, C.

We know that the standard form of the equation of the circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)

Substituting A(5,5) in (1), we get,

⇒ $5^2 + 5^2 + 2a(5) + 2b(5) + c = 0$

⇒ $25 + 25 + 10a + 10b + c = 0$

⇒ $10a + 10b + c + 50 = 0$ ..... (2)

Substituting B(6,4) in (1), we get,

⇒ $6^2 + 4^2 + 2a(6) + 2b(4) + c = 0$

⇒ $36 + 16 + 12a + 8b + c = 0$

⇒ $12a + 8b + c + 52 = 0$ .....(3)

Substituting C(- 2,4) in (1), we get,

⇒ $(- 2)^2 + 4^2 + 2a(- 2) + 2b(4) + c = 0$

⇒ $4 + 16 - 4a + 8b + c = 0$

⇒ $20 - 4a + 8b + c = 0$

⇒ $4a - 8b - c - 20 = 0$ .....(4)

On solving (2), (3) and (4) we get,

⇒ $a = - 2$, $b = - 1$ and $c = - 20$

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2(- 2)x + 2(- 1)y - 20 = 0$

⇒ $x^2 + y^2 - 4x - 2y - 20 = 0$ ..... (5)

Substituting D(7,1) in eq(5) we get,

⇒ $7^2 + 1^2 - 4(7) - 2(1) - 20$

⇒ 49 + 1 - 28 - 2 - 20

⇒ 0

∴ The points (3, - 2), (1,0), (- 1, - 2), (1, - 4) lie on a circle.

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$,

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

Comparing (5) with (1), we get

⇒ Centre = $\left(\frac{-(-4)}{2}, \frac{-(-2)}{2}\right)$

⇒ Centre = (2,1)

⇒ Radius = $\sqrt{2^2 + 1^2 - (-20)}$

⇒ Radius = $\sqrt{25}$

⇒ Radius = 5.

∴ The centre and radius of the circle is (2, 1) and 5.

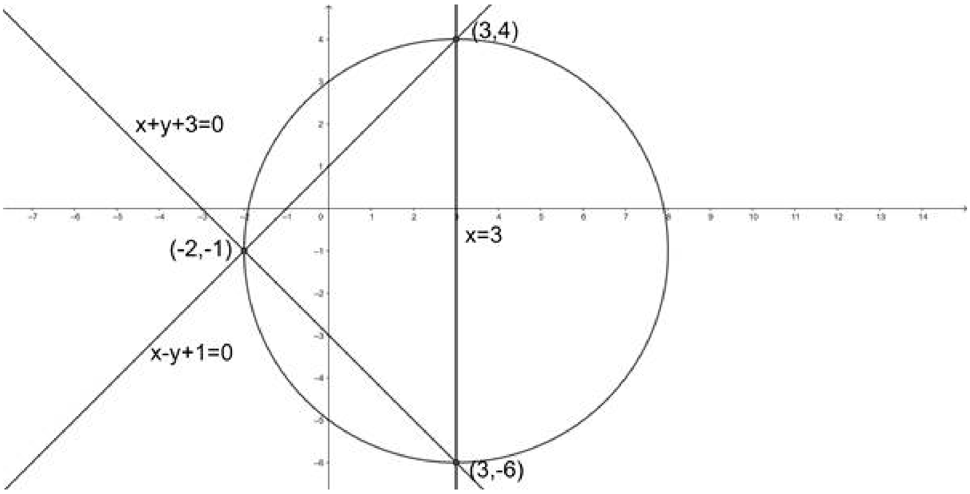
7 A. Question

Find the equation of the circle which circumscribes the triangle formed by the lines:

x + y + 3 = 0, x - y + 1 = 0 and x = 3

Answer

Given that we need to find the equation of the circle formed by the lines:



⇒ x + y + 3 = 0

⇒ x - y + 1 = 0

⇒ x = 3

On solving these lines we get the intersection points A(- 2, - 1), B(3,4), C(3, - 6)

We know that the standard form of the equation of a circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)

Substituting (- 2, - 1) in (1), we get

⇒ $(- 2)^2 + (- 1)^2 + 2a(- 2) + 2b(- 1) + c = 0$

⇒ 4 + 1 - 4a - 2b + c = 0

⇒ 5 - 4a - 2b + c = 0

⇒ 4a + 2b - c - 5 = 0 ..... (2)

Substituting (3,4) in (1), we get

⇒ $3^2 + 4^2 + 2a(3) + 2b(4) + c = 0$

⇒ 9 + 16 + 6a + 8b + c = 0

⇒ 6a + 8b + c + 25 = 0 ..... (3)

Substituting (3, - 6) in (1), we get

⇒ $3^2 + (-6)^2 + 2a(3) + 2b(-6) + c = 0$

⇒ 9 + 36 + 6a - 12b + c = 0

⇒ 6a - 12b + c + 45 = 0 ..... (4)

Solving (2), (3), (4) we get

⇒ a = - 3, b = 1,c = - 15.

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2(-3)x + 2(1)y - 15 = 0$

⇒ $x^2 + y^2 - 6x + 2y - 15 = 0$

∴ The equation of the circle is $x^2 + y^2 - 6x + 2y - 15 = 0$.

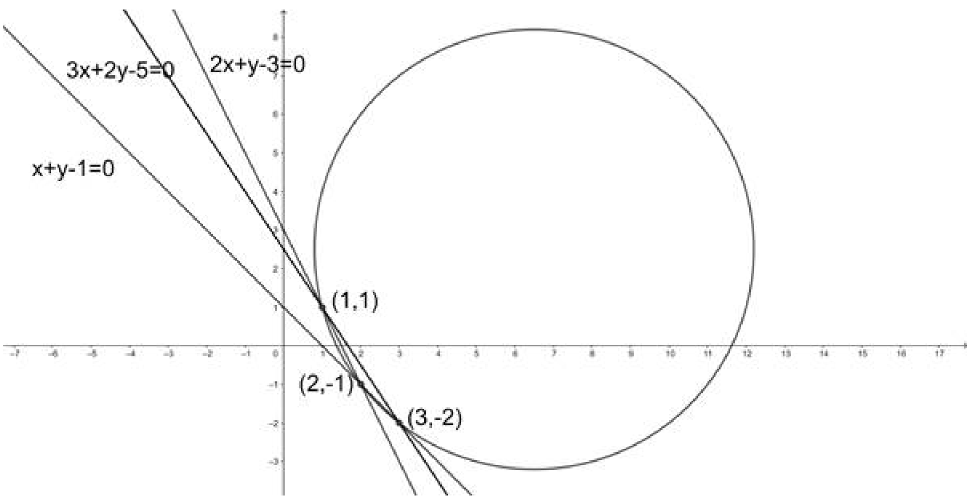
7 B. Question

Find the equation of the circle which circumscribes the triangle formed by the lines:

2x + y - 3 = 0, x + y - 1 = 0 and 3x + 2y - 5 = 0

Answer

Given that we need to find the equation of the circle formed by the lines:



⇒ 2x + y - 3 = 0

⇒ x + y - 1 = 0

⇒ 3x + 2y - 5 = 0

On solving these lines we get the intersection points A(2, - 1), B(3, - 2), C(1,1)

We know that the standard form of the equation of a circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ .....(1)

Substituting (2, - 1) in (1), we get

⇒ $2^2 + (-1)^2 + 2a(2) + 2b(-1) + c = 0$

⇒ $4 + 1 + 4a - 2b + c = 0$

⇒ $4a - 2b + c + 5 = 0$ ..... (2)

Substituting (3, - 2) in (1), we get

⇒ $3^2 + (-2)^2 + 2a(3) + 2b(-2) + c = 0$

⇒ $9 + 4 + 6a - 4b + c = 0$

⇒ $6a - 4b + c + 13 = 0$ ..... (3)

Substituting (1,1) in (1), we get

⇒ $1^2 + 1^2 + 2a(1) + 2b(1) + c = 0$

⇒ $1 + 1 + 2a + 2b + c = 0$

⇒ $2a + 2b + c + 2 = 0$ ..... (4)

Solving (2), (3), (4) we get

⇒ $a = \frac{-13}{2}, b = \frac{-5}{2}, c = 16.$

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2\left(\frac{-13}{2}\right)x + 2\left(\frac{-5}{2}\right)y + 16 = 0$

⇒ $x^2 + y^2 - 13x - 5y + 16 = 0$

∴ The equation of the circle is $x^2 + y^2 - 13x - 5y + 16 = 0$.

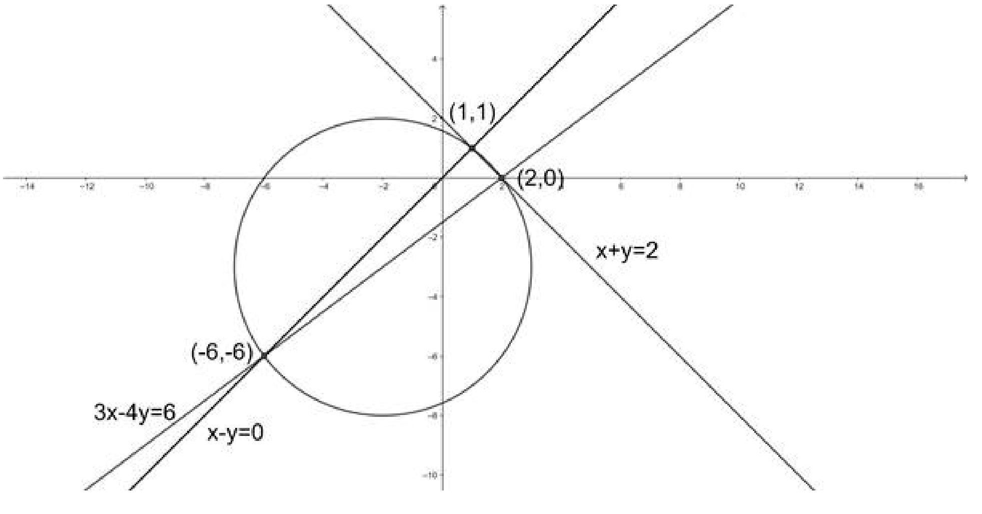
7 C. Question

Find the equation of the circle which circumscribes the triangle formed by the lines:

$x + y = 2$, $3x - 4y = 6$ and $x - y = 0$

Answer

Given that we need to find the equation of the circle formed by the lines:



⇒ $x + y = 2$

⇒ $3x - 4y = 6$

⇒ $x - y = 0$

On solving these lines we get the intersection points A(2,0), B(- 6, - 6), C(1,1)

We know that the standard form of the equation of a circle is given by:

$\Rightarrow x^2 + y^2 + 2ax + 2by + c = 0$ .....(1)

Substituting (2,0) in (1), we get

$\Rightarrow 2^2 + 0^2 + 2a(2) + 2b(0) + c = 0$

$\Rightarrow 4 + 4a + c = 0$

$\Rightarrow 4a + c + 4 = 0$ ..... (2)

Substituting (- 6, - 6) in (1), we get

$\Rightarrow (-6)^2 + (-6)^2 + 2a(-6) + 2b(-6) + c = 0$

$\Rightarrow 36 + 36 - 12a - 12b + c = 0$

$\Rightarrow 12a + 12b - c - 72 = 0$ ..... (3)

Substituting (1,1) in (1), we get

$\Rightarrow 1^2 + 1^2 + 2a(1) + 2b(1) + c = 0$

$\Rightarrow 1 + 1 + 2a + 2b + c = 0$

$\Rightarrow 2a + 2b + c + 2 = 0$ ..... (4)

Solving (2), (3), (4) we get

$\Rightarrow a = 2, b = 3, c = -12.$

Substituting these values in (1), we get

$\Rightarrow x^2 + y^2 + 2(2)x + 2(3)y - 12 = 0$

$\Rightarrow x^2 + y^2 + 4x + 6y - 12 = 0$

$\therefore$ The equation of the circle is $x^2 + y^2 + 4x + 6y - 12 = 0$.

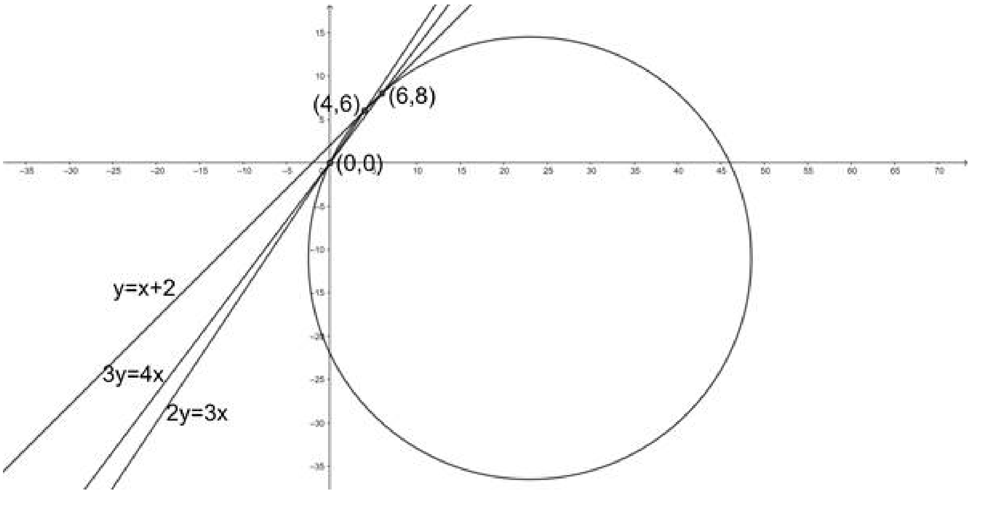
7 D. Question

Find the equation of the circle which circumscribes the triangle formed by the lines:

y = x + 2, 3y = 4x and 2y = 3x

Answer

Given that we need to find the equation of the circle formed by the lines:



$\Rightarrow y = x + 2$

$\Rightarrow 3y = 4x$

$\Rightarrow 2y = 3x$

On solving these lines we get the intersection points A(6,8), B(0,0), C(4,6)

We know that the standard form of the equation of a circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ .....(1)

Substituting (6,8) in (1), we get

⇒ $6^2 + 8^2 + 2a(6) + 2b(8) + c = 0$

⇒ $36 + 64 + 12a + 16b + c = 0$

⇒ $12a + 16b + c + 100 = 0$ ......(2)

Substituting (0,0) in (1), we get

⇒ $0^2 + 0^2 + 2a(0) + 2b(0) + c = 0$

⇒ $0 + 0 + 0a + 0b + c = 0$

⇒ $c = 0$ .....(3)

Substituting (4,6) in (1), we get

⇒ $4^2 + 6^2 + 2a(4) + 2b(6) + c = 0$

⇒ $16 + 36 + 8a + 12b + c = 0$

⇒ $8a + 12b + c + 52 = 0$ ..... (4)

Solving (2), (3), (4) we get

⇒ $a = - 23, b = 11, c = 0$.

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2(- 23)x + 2(11)y + 0 = 0$

⇒ $x^2 + y^2 - 46x + 22y = 0$

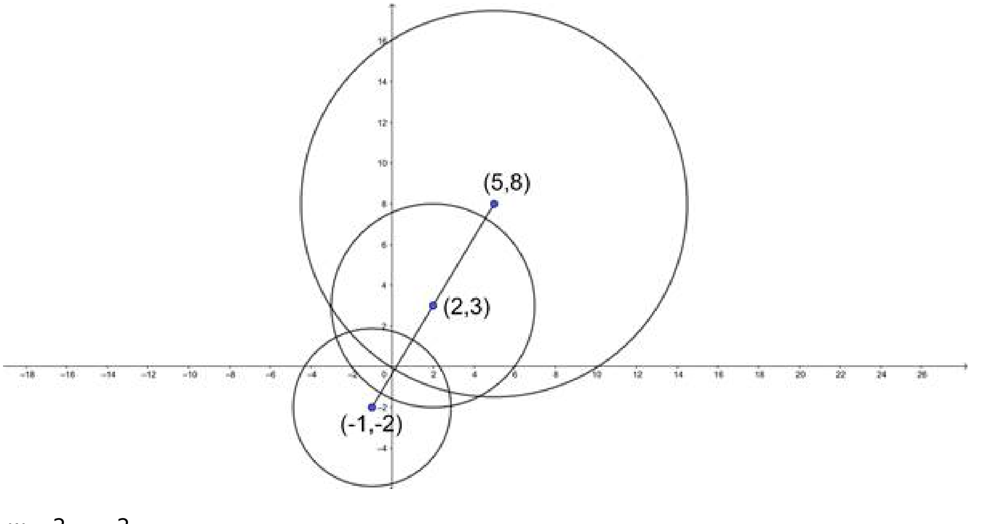
∴ The equation of the circle is $x^2 + y^2 - 46x + 22y = 0$.

8. Question

Prove that the centres of the three circles $x^2 + y^2 – 4x – 6y – 12 = 0$, $x^2 + y^2 + 2x + 4y – 10 = 0$ and $x^2 + y^2 – 10x – 16y – 1 = 0$ are collinear.

Answer

Given circles are:



(i) $x^2 + y^2 - 4x - 6y - 12 = 0$

(ii) $x^2 + y^2 + 2x + 4y - 10 = 0$

(iii) $x^2 + y^2 - 10x - 16y - 1 = 0$

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$ - - (1)

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

Comparing (i) with (1) we get,

⇒ Centre(A) = $\left(\frac{-(-4)}{2}, \frac{-(-6)}{2}\right)$

⇒ A = (2,3)

Comparing (ii) with (1) we get,

⇒ Centre(B) = $\left(\frac{-2}{2}, \frac{-4}{2}\right)$

⇒ B = (- 1, - 2)

Comparing (iii) with (1) we get,

⇒ Centre(C) = $\left(\frac{-(-10)}{2}, \frac{-(-16)}{2}\right)$

⇒ C = (5,8)

We need to show that A, B, and C are collinear.

We find the line passing through any two points and check whether the third point is present on it or not, by substituting the point in the line.

Let us assume the line passes through the points A, B.

We know that the equation of the line passing through the points (x_1,y_1) and (x_2,y_2) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
.

⇒ $y - 3 = \frac{-2-3}{-1-2}(x - 2)$

⇒ $y - 3 = \frac{5}{3}(x - 2)$

⇒ 3(y - 3) = 5(x - 2)

⇒ 3y - 9 = 5x - 10

⇒ 5x - 3y - 1 = 0 ..... (2)

Substituting point C(5,8) in (2), we get

⇒ 5(5) - 3(8) - 1

⇒ 25 - 24 - 1

⇒ 0

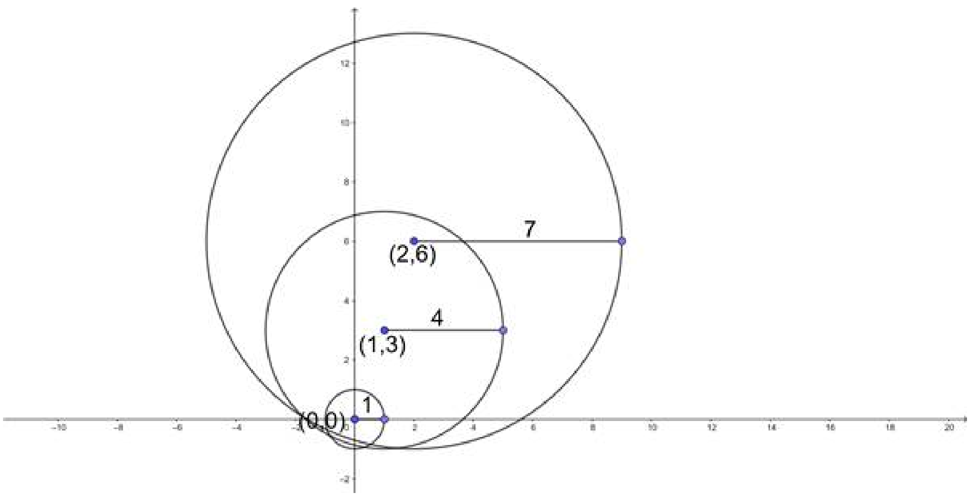
∴ The centres of the circle are collinear.

9. Question

Prove that the radii of the circles $x^2 + y^2 = 1$, $x^2 + y^2 - 2x - 6y - 6 = 0$ and $x^2 + y^2 - 4x - 12y - 9 = 0$ are in A. P.

Answer

Given circles are:



(i) $x^2 + y^2 - 1 = 0$

(ii) $x^2 + y^2 - 2x - 6y - 6 = 0$

(iii) $x^2 + y^2 - 4x - 12y - 9 = 0$

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$ - - (1)

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

Comparing (i) with (1) we get,

⇒ Radius (r_1) = $\sqrt{0^2 + 0^2 - (-1)}$

⇒ $r_1 = 1$

Comparing (ii) with (1) we get,

⇒ Radius (r_2) = $\sqrt{(1)^2 + (3)^2 - (-6)}$

⇒ $r_2 = \sqrt{1 + 9 + 6}$

⇒ $r_2 = \sqrt{16}$

⇒ $r_2 = 4$

Comparing (iii) with (1) we get,

⇒ Radius (r_3) = $\sqrt{(2)^2 + (6)^2 - (-9)}$

⇒ $r_3 = \sqrt{4 + 36 + 9}$

⇒ $r_3 = \sqrt{49}$

⇒ $r_3 = 7$

We need to show r_1, r_2, r_3 are in A.P.

We know that for three numbers a, b, c to be in A.P. The condition to be satisfied is:

⇒ $b = \frac{a + c}{2}$

⇒ $b = \frac{r_1 + r_3}{2}$

⇒ $b = \frac{1 + 7}{2}$

⇒ $b = \frac{8}{2}$

⇒ b = 4

⇒ $b = r_2$

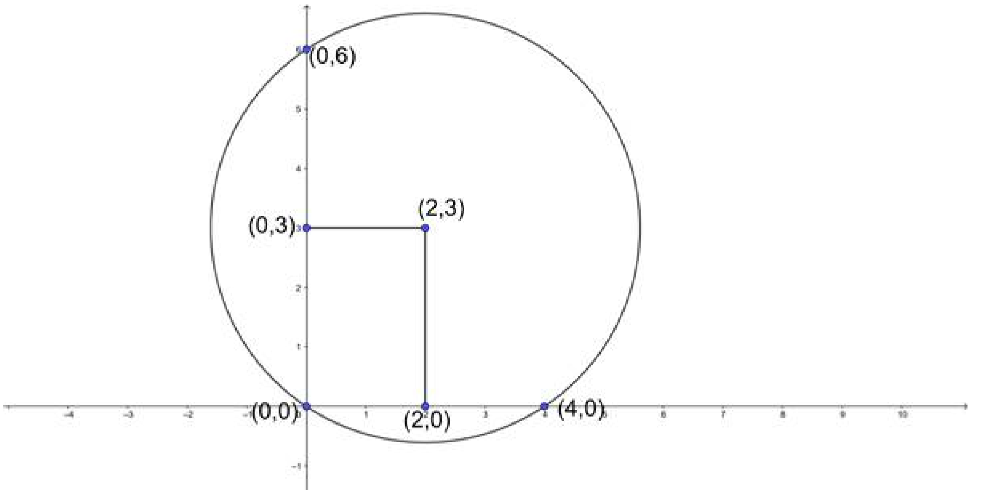
∴ The radii r_1, r_2, r_3 are in A.P.

10. Question

Find the equation of the circle which passes through the origin and cuts off chords of lengths 4 and 6 on the positive side of the x - axis and y - axis respectively.

Answer

Given that we need to find the equation of the circle which passes through the origin and cuts off chords of lengths 4 and 6 on the positive side of the x - axis and y - axis.



Since the circle passes through origin O. So, the points on x and y - axis which are intersected by the circle are A(4, 0) and B(0, 6).

The mid - point of O(0,0) and A(4,0) is C(2,0) and that of O(0,0) and B(0,6) is D(0,3).

Let us assume that P is the centre of the circle.

From the figure, we can see that the line PC is perpendicular bisector of the chord OA and line PD is perpendicular bisector of the chord OB.

We get the centre of the circle to be (2,3).

We have a circle with centre (2,3) and passing through the point (0,0).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(2-0)^2 + (3-0)^2}$

$\Rightarrow r = \sqrt{2^2 + 3^2}$

$\Rightarrow r = \sqrt{4 + 9}$

⇒ $r = \sqrt{13}$ ..... (1)

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x-2)^2 + (y-3)^2 = \left(\sqrt{13}\right)^2$

⇒ $x^2 - 4x + 4 + y^2 - 6y + 9 = 13$

⇒ $x^2 + y^2 - 4x - 6y = 0.$

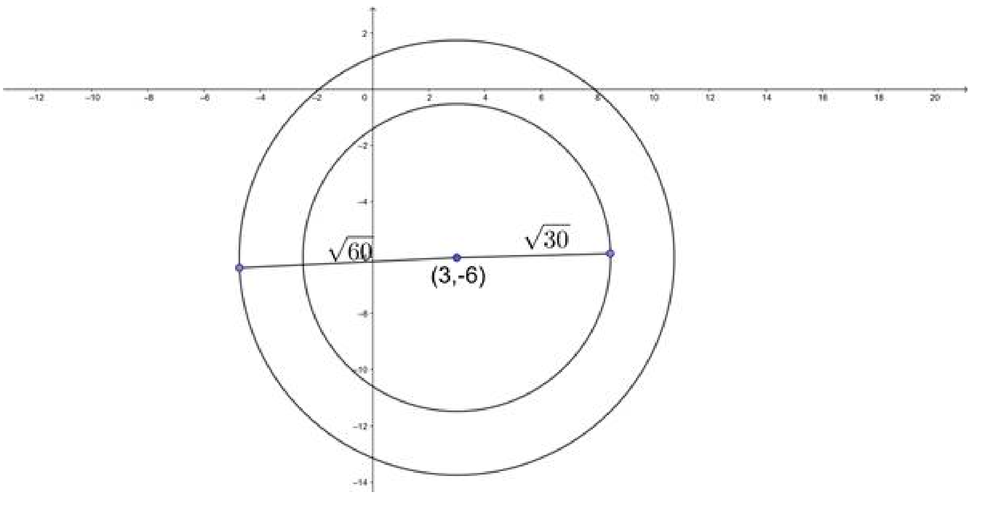
∴ The equation of the circle is $x^2 + y^2 - 4x - 6y = 0.$

11. Question

Find the equation of the circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ and double of its area.

Answer

Given that we need to find the equation of the circle which is concentric with $x^2 + y^2 - 6x + 12y + 15 = 0$ and double its area.



We know that concentric circles will have the same centre.

Let us assume the concentric circle be $x^2 + y^2 - 6x + 12y + c = 0$. .....(ii)

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$ - - (1)

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

Let us assume the radius of the first circle is r_1.

⇒ $r_1 = \sqrt{3^2 + 6^2 - 15}$

⇒ $r_1 = \sqrt{9 + 36 - 15}$

⇒ $r_1 = \sqrt{30}$

Given that the concentric circle's area is double the first circle's area.

Let us assume the radius of the concentric circle be r_2.

We know that the area of the circle is πr^2

Now,

⇒ Area of concentric circle = 2 × area of the first circle

⇒ $\pi r_2^2 = 2 \times \pi r_1^2$

⇒ $r_2^2 = 2 \times (\sqrt{30})^2$

⇒ $r_2^2 = 60$

⇒ $r_2 = \sqrt{60}$

Comparing with (ii) with (1), we get

$\Rightarrow \sqrt{3^2 + 6^2 - c} = \sqrt{60}$

⇒ 9 + 36 - c = 60

⇒ c = - 15

Substituting c value in (ii), we get

⇒ $x^2 + y^2 - 6x + 12y - 15 = 0$

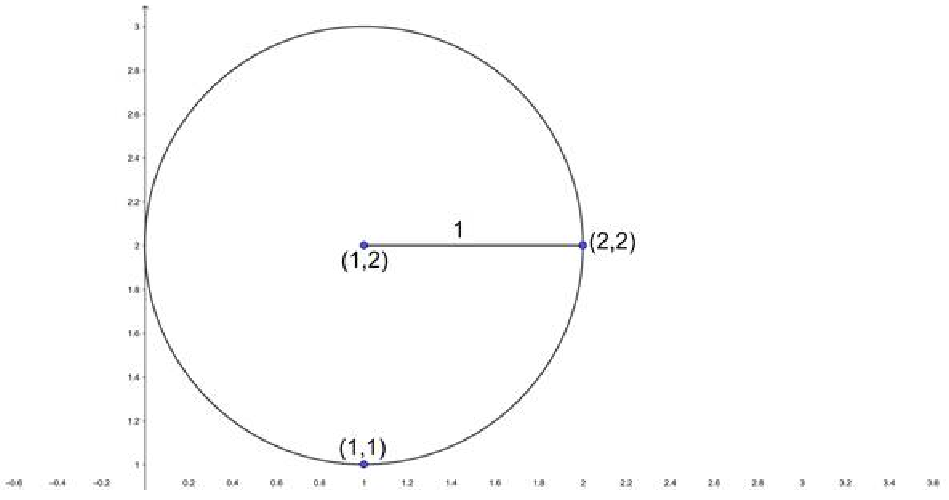
∴ The equation of the concentric circle is $x^2 + y^2 - 6x + 12y - 15 = 0$.

12. Question

Find the equation to the circle which passes through the points (1, 1)(2, 2) and whose radius is 1. Show that there are two such circles.

Answer

Given that we need to find the equation of the circle which passes through the points (1,1), (2,2) and having radius 1.



Let us assume the equation of the circle is:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ .....(1)

We know that the radius of the circle is $\sqrt{a^2 + b^2 - c}$

$\Rightarrow 1 = \sqrt{a^2 + b^2 - c}$

⇒ $a^2 + b^2 - c = 1$ .....(2)

Substituting the point (1,1) in (1) we get,

⇒ $1^2 + 1^2 + 2a(1) + 2b(1) + c = 0$

⇒ 1 + 1 + 2a + 2b + c = 0

⇒ 2a + 2b + c + 2 = 0

$\Rightarrow a + b + \frac{c}{2} = -1$ ..... (3)

Substituting the point (2,2) in (1) we get,

⇒ $2^2 + 2^2 + 2a(2) + 2b(2) + c = 0$

⇒ 4 + 4 + 4a + 4b + c = 0

⇒ 4a + 4b + c + 8 = 0

$\Rightarrow a + b + \frac{c}{4} = -2$ ..... (4)

Subtracting (3) from (4), we get

$\Rightarrow \frac{c}{4} = 1$

$\Rightarrow$ c = 4 ..... (5)

Substituting (5) in (3) we get

$\Rightarrow a + b + \frac{4}{2} = -1$

$\Rightarrow$ a + b + 2 = - 1

$\Rightarrow$ a + b = - 3

$\Rightarrow$ a = - 3 - b ......(6)

Substituting (6) in (2) we get,

$\Rightarrow (- 3 - b)^2 + b^2 - 4 = 1$

$\Rightarrow 9 + b^2 + 6b + b^2 - 5 = 0$

$\Rightarrow 2b^2 + 6b + 4 = 0$

$\Rightarrow b^2 + 3b + 2 = 0$

$\Rightarrow b^2 + 2b + b + 2 = 0$

$\Rightarrow$ b(b + 2) + 1(b + 2) = 0

$\Rightarrow$ (b + 1)(b + 2) = 0

$\Rightarrow$ b + 1 = 0 (or) b + 2 = 0

$\Rightarrow$ b = - 1 (or) b = - 2

For b = - 1, substituting in (6)

$\Rightarrow$ a = - 3 - (- 1)

$\Rightarrow$ a = - 2

For b = - 2, substituting in (6)

$\Rightarrow$ a = - 3 - (- 2)

$\Rightarrow$ a = - 1

Now for a = - 2, b = - 1 and c = 4, the equation of the circle is $x^2 + y^2 - 4x - 2x + 4 = 0$

For a = - 1, b = - 2 and c = 4, the equation of the circle is $x^2 + y^2 - 2x - 4y + 4 = 0$

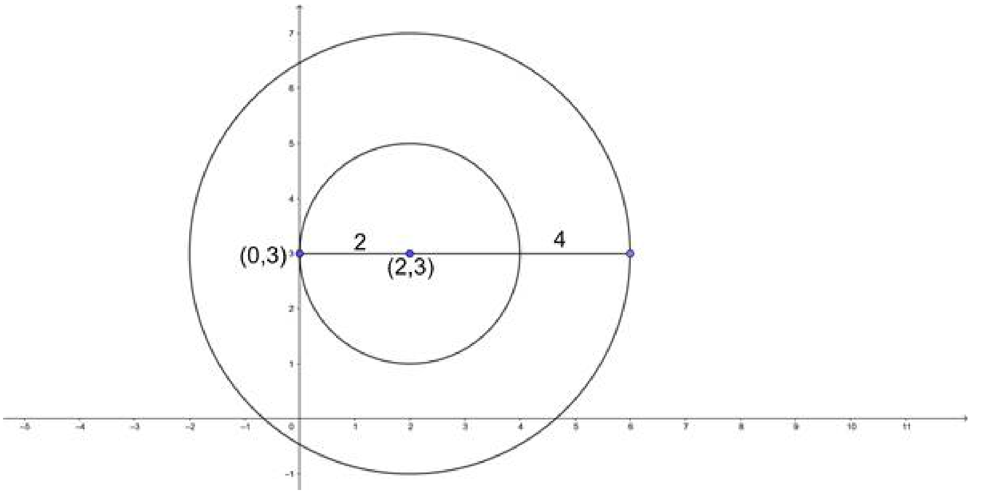
$\therefore$ The equations of the circles are $x^2 + y^2 - 4x - 2y + 4 = 0$ and $x^2 + y^2 - 2x - 4y + 4 = 0$.

13. Question

Find the equation of the circle concentric with $x^2 + y^2 – 4x – 6y – 3 = 0$ and which touches the y - axis.

Answer

Given that we need to find the equation of the circle which is concentric with $x^2 + y^2 - 4x - 6y - 3 = 0$ which touches the y - axis.



We know that the concentric circles will have the same centre.

Let us assume the equation of the concentric circle be $x^2 + y^2 - 4x - 6y + c = 0$. ......- (1)

We know that the value of x is 0 on the y - axis.

Substituting x = 0 in (1), we get

$\Rightarrow 0^2 + y^2 - 4(0) - 6y + c = 0$

$\Rightarrow y^2 - 6y + c = 0$ ......- (2)

We need to get only similar roots on solving the quadratic equation (2), since the circle touches y - axis at only one point.

We know that for a quadratic equation $ax^2 + bx + c = 0$, to have similar roots the condition need to be satisfied is:

$\Rightarrow b^2 - 4ac = 0$

From (2),

$\Rightarrow (-6)^2 - 4(1)(c) = 0$

$\Rightarrow 36 - 4c = 0$

$\Rightarrow 4c = 36$

$\Rightarrow c = \frac{36}{4}$

$\Rightarrow c = 9$ ..... (3)

Substituting (3) in (1), we get

$\Rightarrow x^2 + y^2 - 4x - 6y + 9 = 0$

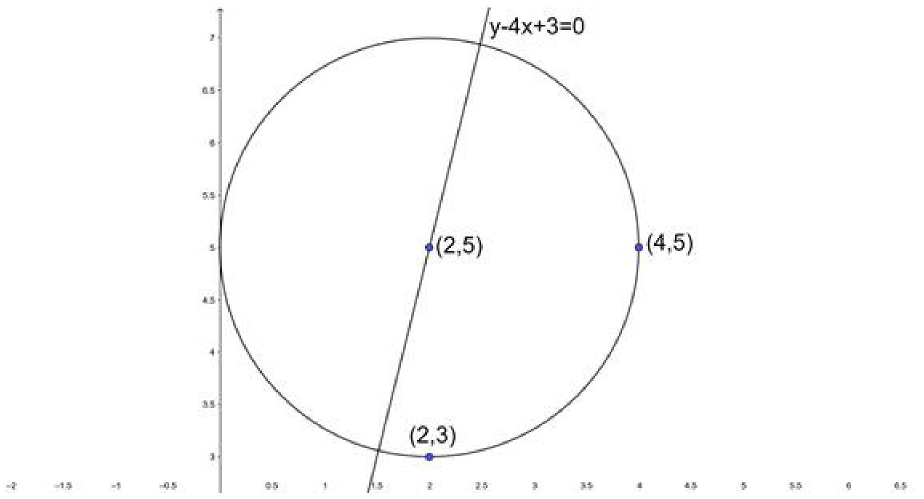
∴ The equation of the concentric circle which touches y - axis is $x^2 + y^2 - 4x - 6y + 9 = 0$.

14. Question

If a circle passes through the point (0, 0), (a, 0), (0, b), then find the coordinates of its centre.

Answer

Given that the circle passes through the points O(0,0), A(a,0) and B(0,b).



Let us first find the length of the sides of the triangle formed by the points OAB.

We know that distance between two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow OA = \sqrt{(0 - a)^2 + (0 - 0)^2}$

$\Rightarrow OA = \sqrt{a^2}$

$\Rightarrow OA = |a|$ .....(1)

$\Rightarrow OB = \sqrt{(0 - 0)^2 + (0 - b)^2}$

$\Rightarrow OB = \sqrt{b^2}$

$\Rightarrow OB = |b|$ .....(2)

$\Rightarrow AB = \sqrt{(a - o)^2 + (0 - b)^2}$

$\Rightarrow AB = \sqrt{a^2 + b^2}$ ..... (3)

Now consider $OA^2 + OB^2$,

$\Rightarrow OA^2 + OB^2 = (|a|)^2 + (|b|)^2$

$\Rightarrow OA^2 + OB^2 = a^2 + b^2$

$\Rightarrow OA^2 + OB^2 = \left(\sqrt{a^2 + b^2}\right)^2$

$\Rightarrow OA^2 + OB^2 = AB^2$

We got ABC is a right angled triangle with AB as the hypotenuse.

We know that the circumcentre of a right-angled triangle is the midpoint of the hypotenuse.

$\Rightarrow$ Centre = $\left(\frac{a+0}{2}, \frac{0+b}{2}\right)$

$\Rightarrow$ Centre = $\left(\frac{a}{2}, \frac{b}{2}\right)$

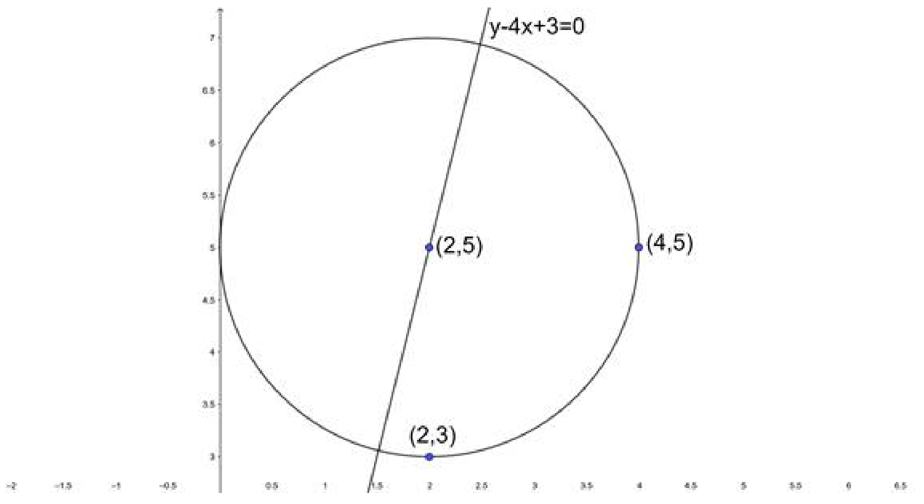
$\therefore$ The coordinates of the centre is $\left(\frac{a}{2}, \frac{b}{2}\right)$.

15. Question

Find the equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line y – 4x + 3 = 0.

Answer

Given that we need to find the equation of the circle which passes through (2,3), (4,5) and has its centre on the line y - 4x + 3 = 0. ..... (1)



We know that the standard form of the equation of the circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ .....(2)

Substituting centre (- a, - b) in (1) we get,

⇒ $-4(-a) + (-b) + 3 = 0$

⇒ $4a - b + 3 = 0$ ......(3)

Substituting (2,3) in (2), we get

⇒ $2^2 + 3^2 + 2a(2) + 2b(3) + c = 0$

⇒ $4 + 9 + 4a + 6b + c = 0$

⇒ $4a + 6b + c + 13 = 0$ ..... (4)

Substituting (4,5) in (2), we get

⇒ $4^2 + 5^2 + 2a(4) + 2b(5) + c = 0$

⇒ $16 + 25 + 8a + 10b + c = 0$

⇒ $8a + 10b + c + 41 = 0$ ..... (5)

Solving (3), (4) and (5) we get,

⇒ $a = -2, b = -5, c = 25$

Substituting these values in (2), we get

⇒ $x^2 + y^2 + 2(-2)x + 2(-5)y + 25 = 0$

⇒ $x^2 + y^2 - 4x - 10y + 25 = 0$

∴ The equation of the circle is $x^2 + y^2 - 4x - 10y + 25 = 0$.

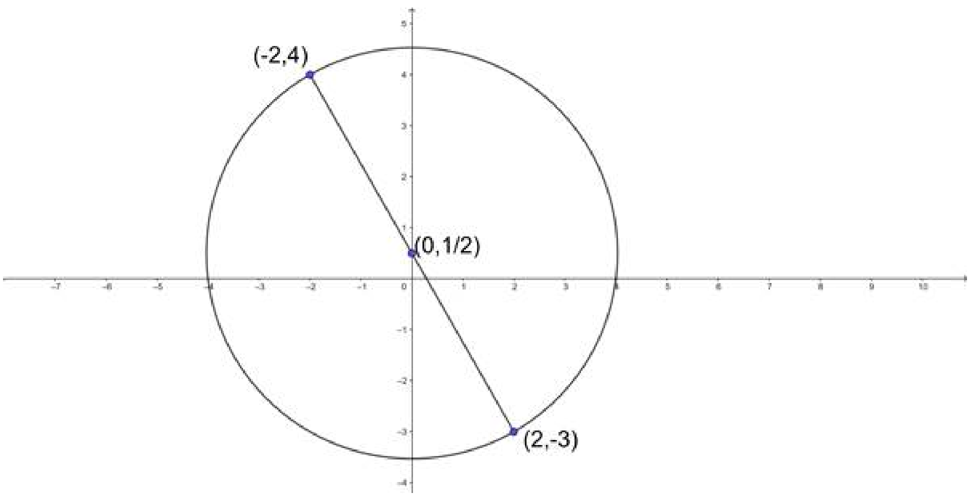
Exercise 24.3

1. Question

Find the equation of the circle, the end points of whose diameter are (2, - 3) and (- 2, 4). Find its centre and radius.

Answer

Given that we need to find the equation of the circle whose end points of a diameter are (2, - 3) and (- 2,4). The figure is given below:



We know that centre is the midpoint of the diameter.

$\Rightarrow$ Centre(C) = $\left(\frac{2-2}{2}, \frac{-3+4}{2}\right)$

$\Rightarrow C = \left(0, \frac{1}{2}\right)$

We have a circle with centre $\left(0, \frac{1}{2}\right)$ and passing through the point (2, - 3).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(2-0)^2 + \left(-3-\frac{1}{2}\right)^2}$

$\Rightarrow r = \sqrt{(2)^2 + \left(\frac{-7}{2}\right)^2}$

$\Rightarrow r = \sqrt{4 + \frac{49}{4}}$

$\Rightarrow r = \sqrt{\frac{65}{4}}$

$\Rightarrow r = \frac{\sqrt{65}}{2}$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x-0)^2 + \left(y-\frac{1}{2}\right)^2 = \left(\frac{\sqrt{65}}{2}\right)^2$

$\Rightarrow x^2 + y^2 - y + \frac{1}{4} = \frac{65}{4}$

$\Rightarrow 4x^2 + 4y^2 - 4y + 1 = 65$

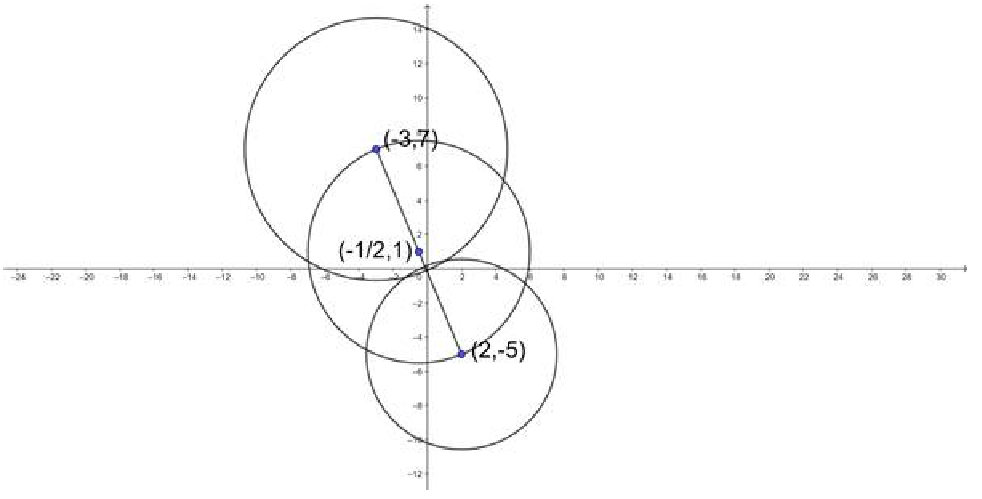
∴The equation of the circle is $4x^2 + 4y^2 - 4y - 64 = 0$ or $\mathbf{x^2 + y^2 - y - 16 = 0}$

2. Question

Find the equation of the circle the end points of whose diameter are the centres of the circles $x^2 + y^2 + 6x - 14y - 1 = 0$ and $x^2 + y^2 - 4x + 10y - 2 = 0$.

Answer

Given that we need to find the equation of the circle whose end points of a diameter are the centres of the circles



$x^2 + y^2 + 6x - 14y - 1 = 0$ .... - (i) and

$x^2 + y^2 - 4x + 10y - 2 = 0$ ...... - (ii)

Let us assume A and B are the centres of the 1st and 2nd circle.

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$ ...... - (1)

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

Comparing (i) with (1) we get,

⇒ Centre(A) = $\left(\frac{-6}{2}, \frac{-(-14)}{2}\right)$

⇒ A = (- 3,7)

Comparing (ii) with (1) we get,

⇒ Centre(B) = $\left(\frac{-(-4)}{2}, \frac{-10}{2}\right)$

⇒ B = (2, - 5)

We know that the centre is the mid - point of the diameter.

⇒ Centre(C) = $\left(\frac{-3+2}{2}, \frac{7-5}{2}\right)$

⇒ $C = \left(\frac{-1}{2}, 1\right)$

We have a circle with centre $\left(\frac{-1}{2}, 1\right)$ and passing through the point (2, - 5).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

⇒ $r = \sqrt{\left(\frac{-1}{2} - 2\right)^2 + (1 - (-5))^2}$

⇒ $r = \sqrt{\left(\frac{-5}{2}\right)^2 + (6)^2}$

$\Rightarrow r = \sqrt{\frac{25}{4} + 36}$

$\Rightarrow r = \sqrt{\frac{169}{4}}$

$\Rightarrow r = \frac{13}{2}$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow \left(x - \left(-\frac{1}{2}\right)\right)^2 + (y - 1)^2 = \left(\frac{13}{2}\right)^2$

$\Rightarrow x^2 + x + \frac{1}{4} + y^2 - 2y + 1 = \frac{169}{4}$

$\Rightarrow x^2 + y^2 + x - 2y - 41 = 0$

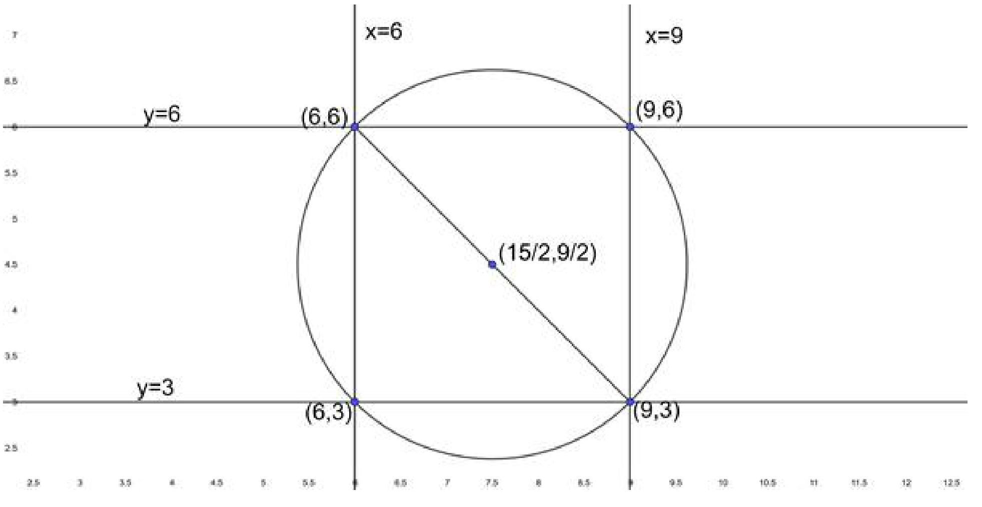
∴The equation of the circle is $x^2 + y^2 + x - 2y - 41 = 0$.

3. Question

The sides of a squares are x = 6, x = 9, y = 3 and y = 6. Find the equation of a circle drawn on the diagonal of the square as its diameter.

Answer

Given that we need to find the equation of the circle with the diagonal of a square as diameter.



It is also told that x = 6, x = 9, y = 3, and y = 6 are the sides of a square.

Let us assume A,B,C,D are the vertices of the square. On solving the lines, we get the vertices as:

$\Rightarrow$ A = (6,3)

$\Rightarrow$ B = (9,3)

$\Rightarrow$ C = (9,6)

$\Rightarrow$ D = (6,6)

Since the diagonal of the square is the diameter of the circle, the circle circumscribes the square.

So, taking any diagonal as diameter gives the same equation of the circle.

Let us assume diagonal BD as the diameter.

We know that the centre is the mid - point of the diameter.

$\Rightarrow$ Centre(O) = $\left(\frac{6+9}{2}, \frac{3+6}{2}\right)$

$\Rightarrow$ O = $\left(\frac{15}{2}, \frac{9}{2}\right)$

We have a circle with centre $\left(\frac{15}{2}, \frac{9}{2}\right)$ and passing through the point (6,3).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$$\Rightarrow r = \sqrt{\left(\frac{15}{2} - 6\right)^2 + \left(\frac{9}{2} - 3\right)^2}$$

$$\Rightarrow r = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$$

$$\Rightarrow r = \sqrt{\frac{9}{4} + \frac{9}{4}}$$

$$\Rightarrow r = \sqrt{\frac{9}{2}}$$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$$\Rightarrow \left(x - \frac{15}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \left(\sqrt{\frac{9}{2}}\right)^2$$

$$\Rightarrow x^2 - 15x + \frac{225}{4} + y^2 - 9y + \frac{81}{4} = \frac{9}{2}$$

$\Rightarrow x^2 + y^2 - 15x - 9y + 72 = 0$

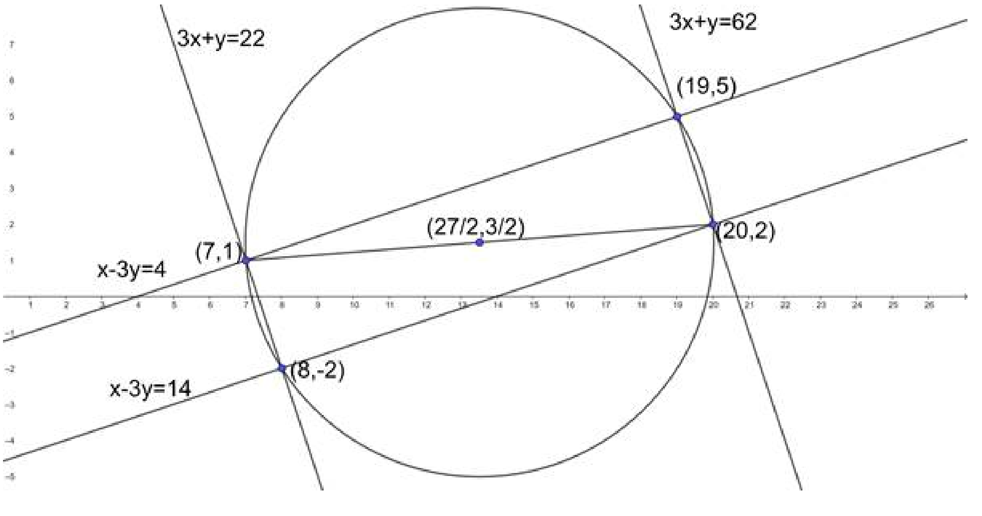
∴The equation of the circle is $x^2 + y^2 - 15x - 9y + 72 = 0$.

4. Question

Find the equation of the circle circumscribing the rectangle whose sides are x – 3y = 4, 3x + y = 22, x – 3y = 14 and 3x + y = 62.

Answer

Given that we need to find the equation of the circle which circumscribes the rectangle.



It is also told that x - 3y = 4, 3x + y = 22, x - 3y = 14 and 3x + y = 62 are the sides of a rectangle.

Let us assume A,B,C,D are the vertices of the rectangle. On solving the lines, we get the vertices as:

⇒ A = (7,1)

⇒ B = (8, - 2)

⇒ C = (20,2)

⇒ D = (19,5)

Since the circle circumscribes the rectangle, the diagonal of the rectangle will be the diameter of the circle.

So, taking any diagonal as diameter gives the same equation of the circle.

Let us assume diagonal AC as the diameter.

We know that the centre is the mid - point of the diameter.

⇒ Centre(C) = $\left(\frac{7+20}{2}, \frac{1+2}{2}\right)$

$\Rightarrow C = \left(\frac{27}{2}, \frac{3}{2}\right)$

We have a circle with a centre $\left(\frac{27}{2}, \frac{3}{2}\right)$ and passing through the point (7,1).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$$\Rightarrow r = \sqrt{\left(\frac{27}{2} - 7\right)^2 + \left(\frac{3}{2} - 1\right)^2}$$

$$\Rightarrow r = \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$\Rightarrow r = \sqrt{\frac{169}{4} + \frac{1}{4}}$$

$$\Rightarrow r = \sqrt{\frac{170}{4}}$$

$$\Rightarrow r = \frac{\sqrt{170}}{2}$$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$$\Rightarrow \left(x - \frac{27}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \left(\frac{\sqrt{170}}{2}\right)^2$$

$$\Rightarrow x^2 - 27x + \frac{729}{4} + y^2 - 3y + \frac{9}{4} = \frac{170}{4}$$

⇒ $x^2 + y^2 - 27x - 3y + 142 = 0$

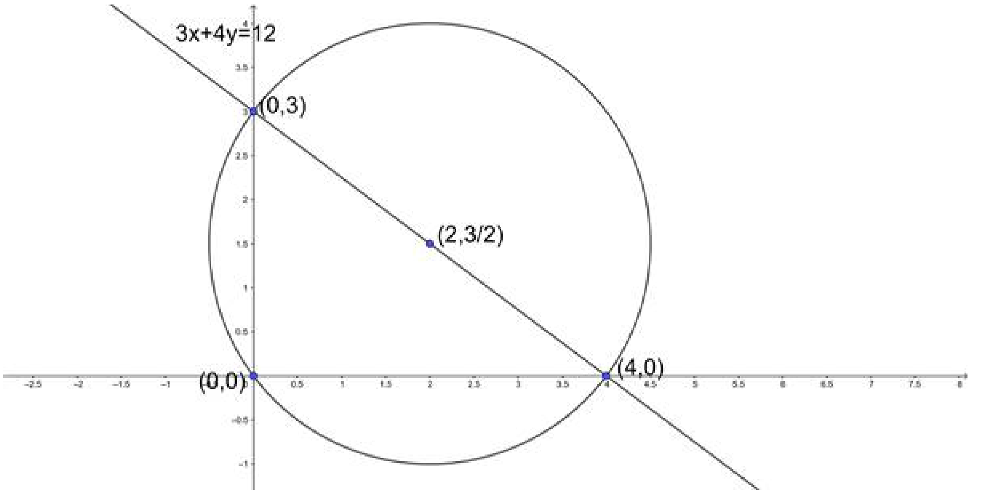
∴The equation of the circle is $x^2 + y^2 - 27x - 3y + 142 = 0$.

5. Question

Find the equation of the circle passing through the origin and the points where the line 3x + 4y = 12 meets the axes of coordinates.

Answer

Given that we need to find the equation of the circle passing through the origin and the points where the line 3x + 4y = 12 meets the axes of co - ordinates.



Let us first find the points at which the line meets the axes.

The value of x is 0 on meeting the y - axis. So,

⇒ 3(0) + 4y = 12

⇒ 4y = 12

⇒ y = 3

The point is A(0,3)

The value of y is 0 on meeting the x - axis. So,

⇒ 3x + 4(0) = 12

⇒ 3x = 12

⇒ x = 4

The point is B(4,0)

We have the circle passing through the points O(0,0), A(0,3) and B(4,0).

We know that the standard form of the equation of the circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)

Substituting O(0,0) in (1), we get,

⇒ $0^2 + 0^2 + 2a(0) + 2b(0) + c = 0$

⇒ c = 0 ..... (2)

Substituting A(0,3) in (1), we get,

⇒ $0^2 + 3^2 + 2a(0) + 2b(3) + c = 0$

⇒ 9 + 6b + c = 0

⇒ 6b + c + 9 = 0 ..... (3)

Substituting B(4,0) in (1), we get,

⇒ $4^2 + 0^2 + 2a(4) + 2b(0) + c = 0$

⇒ 16 + 8a + c = 0

⇒ 8a + c + 16 = 0 ..... (4)

On solving (2), (3) and (4) we get,

$\Rightarrow a = -2, b = \frac{-3}{2}$ and $c = 0$

Substituting these values in (1), we get

$\Rightarrow x^2 + y^2 + 2(-2)x + 2\left(\frac{-3}{2}\right)y + 0 = 0$

$\Rightarrow x^2 + y^2 - 4x - 3y = 0$

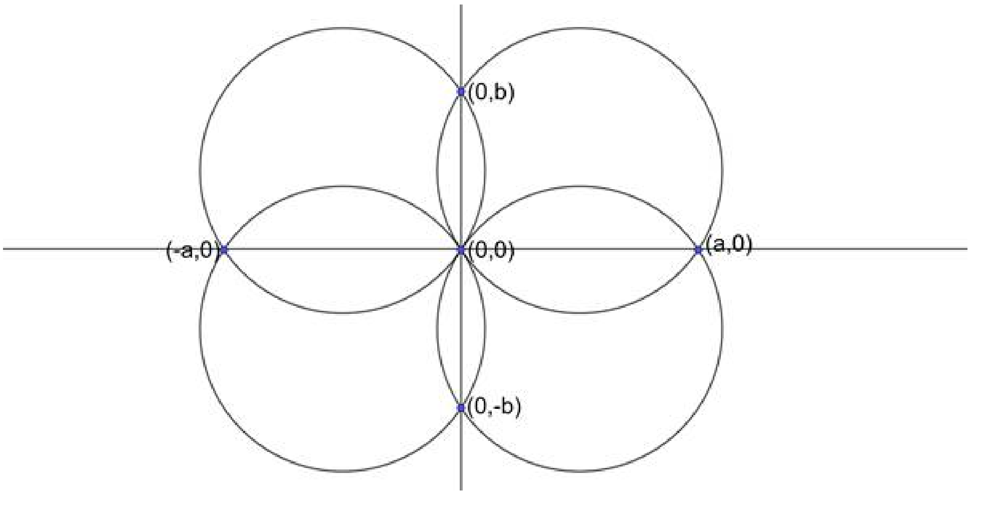
∴The equation of the circle is $x^2 + y^2 - 4x - 3y = 0$.

6. Question

Find the equation of the circle which passes through the origin and cuts off intercepts a and b respectively from x and y - axes.

Answer

Given that we need to find the equation of the circle passing through the origin and cuts off intercepts a and b from x and y - axes.



Since the circle has intercept a from x - axis the circle must pass through (a,0) and (- a,0) as it already passes through the origin.

Since the circle has intercept b from x - axis, the circle must pass through (0,b) and (0, - b) as it already passes through the origin.

Let us assume the circle passing through the points O(0,0), A(a,0) and B(0,b).

We know that the standard form of the equation of the circle is given by:

$\Rightarrow x^2 + y^2 + 2fx + 2gy + c = 0$ ..... (1)

Substituting O(0,0) in (1), we get,

$\Rightarrow 0^2 + 0^2 + 2f(0) + 2g(0) + c = 0$

$\Rightarrow c = 0$ ..... (2)

Substituting A(a,0) in (1), we get,

$\Rightarrow a^2 + 0^2 + 2f(a) + 2g(0) + c = 0$

$\Rightarrow a^2 + 2fa + c = 0$ ..... (3)

Substituting B(0,b) in (1), we get,

$\Rightarrow 0^2 + b^2 + 2f(0) + 2g(b) + c = 0$

$\Rightarrow b^2 + 2gb + c = 0$ ..... (4)

On solving (2), (3) and (4) we get,

$\Rightarrow f = \frac{-a}{2}, b = \frac{-b}{2}$ and $c = 0$

Substituting these values in (1), we get

$\Rightarrow x^2 + y^2 + 2\left(\frac{-a}{2}\right)x + 2\left(\frac{-b}{2}\right)y + 0 = 0$

$\Rightarrow x^2 + y^2 - ax - by = 0$

Similarly, we get the equation $x^2 + y^2 + ax + by = 0$ for the circle passing through the points (0,0), (- a,0), (0, - b).

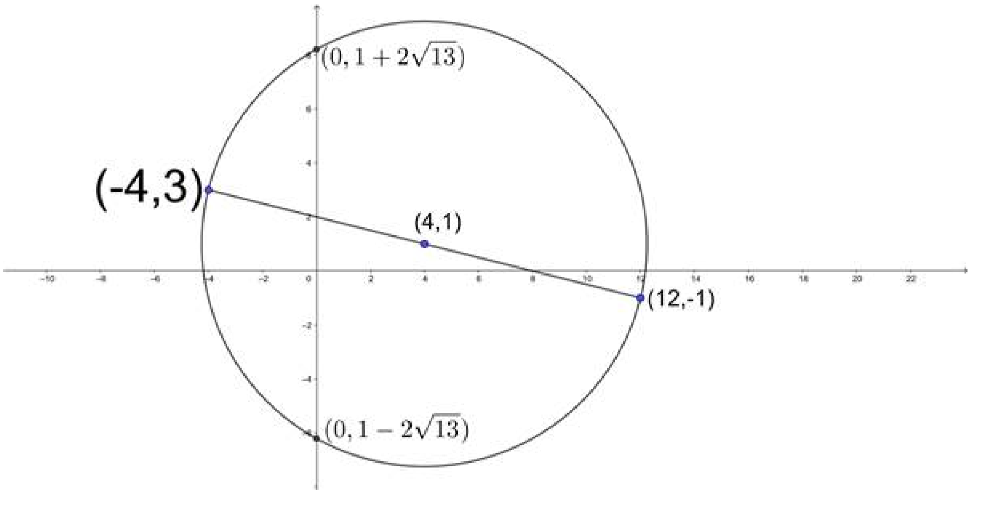
∴The equations of the circles are $x^2 + y^2 \pm ax \pm by = 0$.

7. Question

Find the equation of the circle whose diameter is the line segment joining (- 4, 3) and (12, - 1). Find also the intercept made by it on the y-axis.

Answer

Given that we need to find the equation of the circle whose end points of a diameter are (- 4,3) and (12, - 1).



We know that the centre is the mid - point of the diameter.

$\Rightarrow$ Centre(C) = $\left(\frac{-4+12}{2}, \frac{3-1}{2}\right)$

$\Rightarrow$ C = (4,1)

We have a circle with centre (4,1) and passing through the point (- 4,3).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(4 - (-4))^2 + (1 - 3)^2}$

$\Rightarrow r = \sqrt{(8)^2 + (-2)^2}$

$\Rightarrow r = \sqrt{64 + 4}$

$\Rightarrow r = \sqrt{68}$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - 4)^2 + (y - 1)^2 = (\sqrt{68})^2$

$\Rightarrow x^2 - 8x + 16 + y^2 - 2y + 1 = 68$

$\Rightarrow x^2 + y^2 - 8x - 2y - 51 = 0$ ..... (1)

To find the y - intercept, we need the find the points at which the circle intersects the y - axis.

We know that x = 0 on y - axis. Substituting x = 0 in (1) we get

$\Rightarrow 0^2 + y^2 - 8(0) - 2y - 51 = 0$

$\Rightarrow y^2 - 2y - 51 = 0$

$\Rightarrow y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-51)}}{2(1)}$

$\Rightarrow y = \frac{2 \pm \sqrt{4 + 204}}{2}$

$\Rightarrow y = 1 \pm 2\sqrt{13}$

∴The y - intercepts are $1 \pm 2\sqrt{13}$.

8. Question

The abscissae of the two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation of the circle with AB as diameter. Also, find its radius.

Answer

Given that points, we need to find the equation of the circle whose ends of the diameter are A and B.

It is also told that the abscissae of two points A and B are the roots of $x^2 + 2ax - b^2 = 0$ and ordinates are the roots of $x^2 + 2px - q^2 = 0$.

Let us first find the roots of each quadratic equation.

For $x^2 + 2ax - b^2 = 0$

$\Rightarrow x = \frac{-2a \pm \sqrt{(2a)^2 - 4(1)(-b^2)}}{2(1)}$

$\Rightarrow x = \frac{-2a \pm \sqrt{4(a^2 + b^2)}}{2}$

$\Rightarrow x = -a \pm \sqrt{a^2 + b^2}$ ..... (1)

For $x^2 + 2px - q^2 = 0$

$\Rightarrow x = \frac{-2p \pm \sqrt{(2p)^2 - 4(1)(-q^2)}}{2(1)}$

$\Rightarrow x = \frac{-2p \pm \sqrt{4(p^2 + q^2)}}{2}$

$\Rightarrow x = -p \pm \sqrt{p^2 + q^2}$ ..... (2)

From (1) and (2) we get,

$\Rightarrow A = \left(-a + \sqrt{a^2 + b^2}, -p + \sqrt{p^2 + q^2}\right)$

$\Rightarrow B = \left(-a - \sqrt{a^2 + b^2}, -p - \sqrt{p^2 + q^2}\right)$

We know that the centre is the mid - point of the diameter.

$\Rightarrow \text{Centre(C)} = \left(\frac{-a + \sqrt{a^2 + b^2} - a - \sqrt{a^2 + b^2}}{2}, \frac{-p + \sqrt{p^2 + q^2} - p - \sqrt{p^2 + q^2}}{2}\right)$

$\Rightarrow$ C = (- a, - p)

We have a circle with centre (- a, - p) and passing through the point $\left(-a-\sqrt{a^2+b^2}, -p-\sqrt{p^2+q^2}\right)$.

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{\left(-a-\left(-a-\sqrt{a^2+b^2}\right)\right)^2+\left(-p-\left(-p-\sqrt{p^2+q^2}\right)\right)^2}$

$\Rightarrow r = \sqrt{\left(\sqrt{a^2+b^2}\right)^2+\left(\sqrt{p^2+q^2}\right)^2}$

$\Rightarrow r = \sqrt{a^2+b^2+p^2+q^2}$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x-(-a))^2+(y-(-p))^2 = \left(\sqrt{a^2+b^2+p^2+q^2}\right)^2$

$\Rightarrow x^2 + 2ax + a^2 + y^2 + 2py + p^2 = a^2 + b^2 + p^2 + q^2$

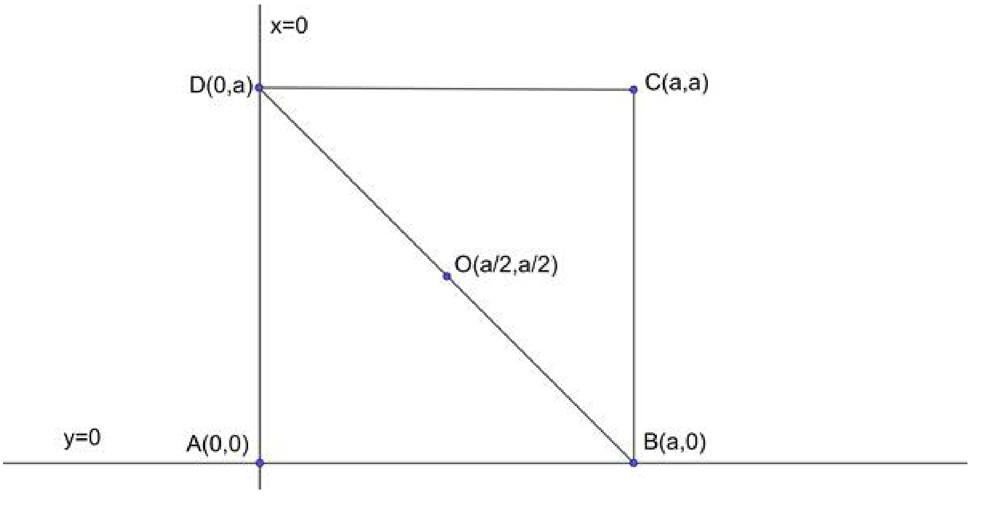
$\Rightarrow x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

$\therefore$The equation of the circle is $x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$.

9. Question

ABCD is a square whose side is a; taking AB and AD as axes, prove that the equation of the circle circumscribing the square is $x^2 + y^2 – a(x + y) = 0$.

Answer



Given that we need to find the equation of the circle which circumscribes the square ABCD of side a.

It is also told that AB and AD are assumed as x and y - axes.

Assuming A as origin, We get the points B(a,0) and D(0,a).

So, we need to find the circle which is passing through the points A(0,0), B(a,0) and D(0,a).

We know that the standard form of the equation of the circle is given by:

$\Rightarrow x^2 + y^2 + 2fx + 2gy + c = 0$ ..... (1)

Substituting A(0,0) in (1), we get,

⇒ $0^2 + 0^2 + 2f(0) + 2g(0) + c = 0$

⇒ $c = 0$ ..... (2)

Substituting B(a,0) in (1), we get,

⇒ $a^2 + 0^2 + 2f(a) + 2g(0) + c = 0$

⇒ $a^2 + 2fa + c = 0$ ..... (3)

Substituting D(0,a) in (1), we get,

⇒ $0^2 + a^2 + 2f(0) + 2g(a) + c = 0$

⇒ $a^2 + 2ga + c = 0$ ..... (4)

On solving (2), (3) and (4) we get,

⇒ $f = \frac{-a}{2}, b = \frac{-a}{2}$ and $c = 0$

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2\left(\frac{-a}{2}\right)x + 2\left(\frac{-a}{2}\right)y + 0 = 0$

⇒ $x^2 + y^2 - ax - ay = 0$

⇒ $x^2 + y^2 - a(x + y) = 0$

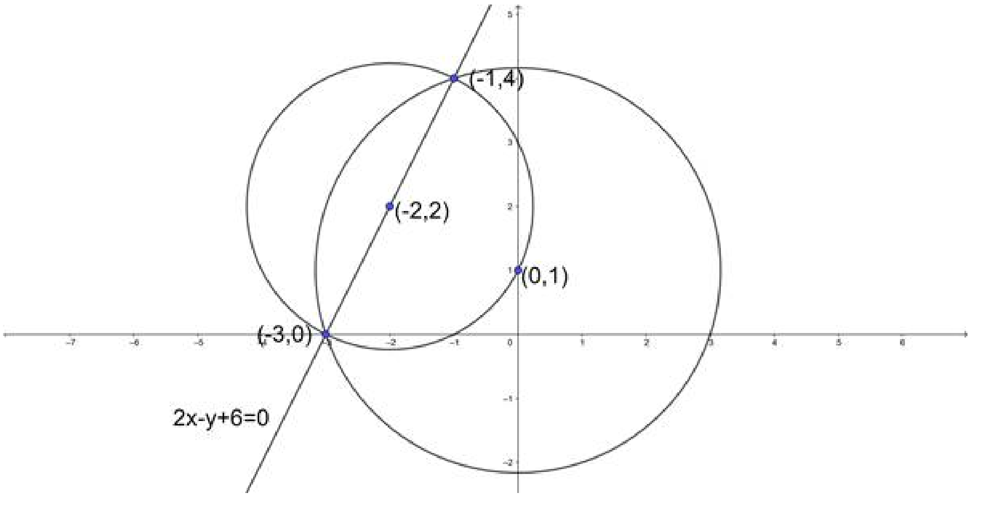
∴Thus proved.

10. Question

The line $2x - y + 6 = 0$ meets the circle $x^2 + y^2 - 2y - 9 = 0$ at A and B. Find the equation of the circle on AB as diameter.

Answer

Given that we need to find the equation of the circle whose diameter is AB.



It is also told that the points A and B are the intersection points when the line $2x - y + 6 = 0$ meets the circle

$x^2 + y^2 - 2y - 9 = 0$. ..... - (1)

Let us first find the points A and B.

From the equation of the line:

⇒ $2x - y + 6 = 0$

⇒ $y = 2x + 6$ ..... (2)

Substituting (2) in (1), we get

⇒ $x^2 + (2x + 6)^2 - 2(2x + 6) - 9 = 0$

⇒ $x^2 + 4x^2 + 24x + 36 - 4x - 12 - 9 = 0$

⇒ $5x^2 + 20x + 15 = 0$

⇒ $5x^2 + 5x + 15x + 15 = 0$

⇒ $5x(x + 1) + 15(x + 1) = 0$

⇒ $(5x + 15)(x + 1) = 0$

⇒ $5x + 15 = 0$ or $x + 1 = 0$

⇒ $5x = - 15$ or $x = - 1$

⇒ $x = - 3$ or $x = - 1$

For $x = - 3$, from (2)

⇒ $y = 2(- 3) + 6$

⇒ $y = - 6 + 6$

⇒ $y = 0$

For $x = - 1$, from (2)

⇒ $y = 2(- 1) + 6$

⇒ $y = - 2 + 6$

⇒ $y = 4$

The points are A(- 3,0) and B(- 1,4)

We know that centre is the mid - point of the diameter.

⇒ Centre(C) = $\left(\frac{-3-1}{2}, \frac{0+4}{2}\right)$

⇒ C = (- 2,2)

We have circle with centre (- 2,2) and passing through the point (- 1,4).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

⇒ $r = \sqrt{(-2-(-1))^2 + (2-4)^2}$

⇒ $r = \sqrt{(-1)^2 + (-2)^2}$

⇒ $r = \sqrt{1 + 4}$

⇒ $r = \sqrt{5}$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

⇒ $(x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

⇒ $(x-(-2))^2 + (y-2)^2 = (\sqrt{5})^2$

⇒ $x^2 + 4x + 4 + y^2 - 4y + 4 = 5$

⇒ $x^2 + y^2 + 4x - 4y + 3 = 0$

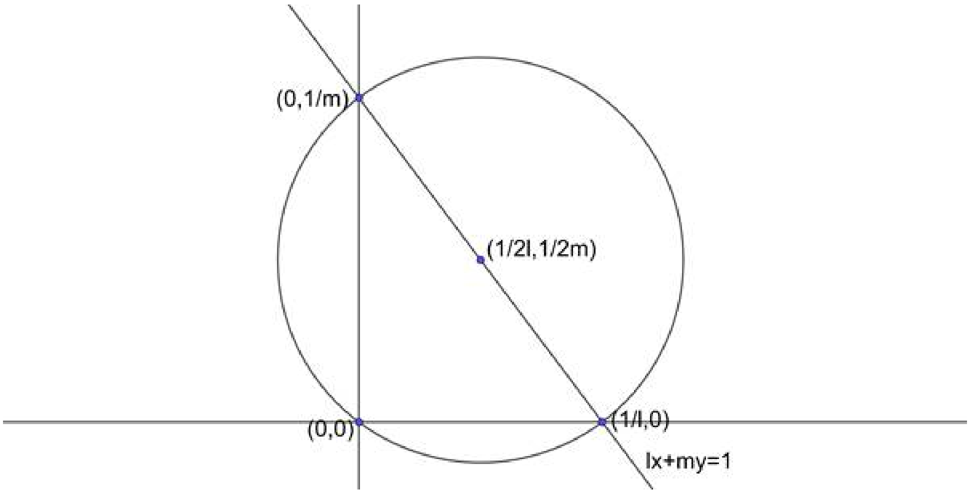
∴The equation of the circle is $x^2 + y^2 + 4x - 4y + 3 = 0$.

11. Question

Find the equation of the circle which circumscribes the triangle formed by the lines x = 0, y = 0 and lx + my = 1.

Answer

Given that we need to find the equation of the circle that circumscribes the triangle formed by the lines x = 0, y = 0, and lx + my = 1.



Let us assume A, B, C are the vertices of the triangle.

On solving the lines we get,

⇒ A = (0,0)

⇒ B = $\left(0,\frac{1}{m}\right)$

⇒ C = $\left(\frac{1}{l},0\right)$

We have the circle passing through the points A(0,0), B$\left(0,\frac{1}{m}\right)$ and C$\left(\frac{1}{l},0\right)$.

We know that the standard form of the equation of the circle is given by:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$ ..... (1)

Substituting A(0,0) in (1), we get,

⇒ $0^2 + 0^2 + 2a(0) + 2b(0) + c = 0$

⇒ c = 0 ..... (2)

Substituting B$\left(0,\frac{1}{m}\right)$ in (1), we get,

⇒ $0^2 + \left(\frac{1}{m}\right)^2 + 2a(0) + 2b\left(\frac{1}{m}\right) + c = 0$

⇒ $\frac{1}{m^2} + \frac{2b}{m} + c = 0$

⇒ $cm^2 + 2mb + 1 = 0$ ..... - (3)

Substituting C$\left(\frac{1}{l},0\right)$ in (1), we get,

⇒ $\left(\frac{1}{l}\right)^2 + 0^2 + 2a\left(\frac{1}{l}\right) + 2b(0) + c = 0$

$\Rightarrow \frac{1}{l^2} + \frac{2a}{l} + c = 0$

$\Rightarrow cl^2 + 2al + 1 = 0$ ..... (4)

On solving (2), (3) and (4) we get,

$\Rightarrow a = \frac{-1}{2l}, b = \frac{-1}{2m}$ and $c = 0$

Substituting these values in (1), we get

$\Rightarrow x^2 + y^2 + 2\left(\frac{-1}{2l}\right)x + 2\left(\frac{-1}{2m}\right)y + 0 = 0$

$\Rightarrow x^2 + y^2 - \frac{x}{l} - \frac{y}{m} = 0$

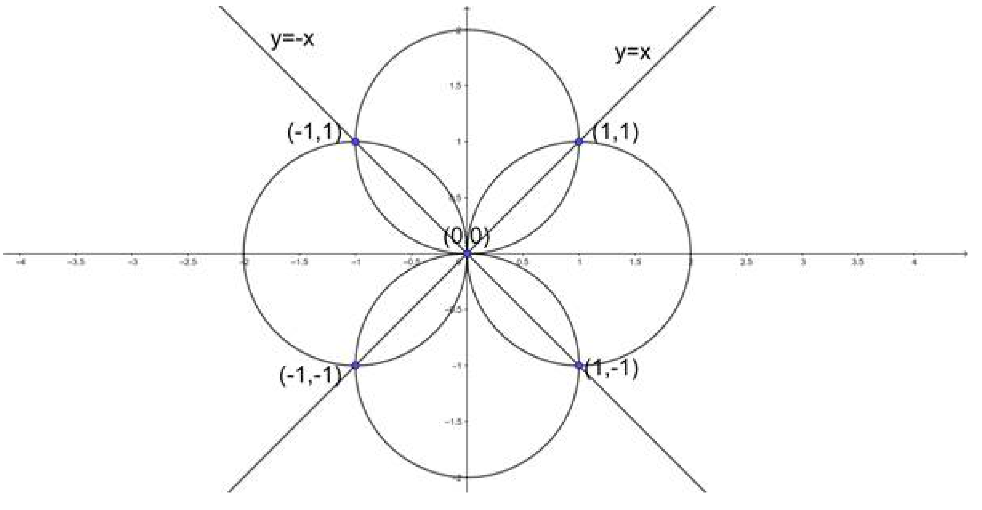
∴The equation of the circle is $x^2 + y^2 - \frac{x}{l} - \frac{y}{m} = 0$.

12. Question

Find the equations of the circles which pass through the origin and cut off equal chords of √2 units from the lines y = x and y = - x.

Answer

We need to find the equations of the circles which pass through the origin and having chords of $\sqrt{2}$ units from the lines y = x and y = - x.



From the figure, we can see that the chords of length √2 units from origin O exists at points A(1,1), B(1, - 1), C(- 1,1) and D(- 1, - 1).

Let us find the distances AB, AC, BD, CD.

We know that distance between two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow AB = \sqrt{(1-1)^2 + (1-(-1))^2}$

$\Rightarrow AB = \sqrt{0^2 + 2^2}$

$\Rightarrow AB = 2$

Similarly AD = BC = CD = 2 units.

We have got right-angled triangles OAB, OAD, OBC, OCD.

We need to find the circle that circumscribes these circles.

We know that the circumcentre of a right-angled triangle is the mid - point of the hypotenuse.

$\Rightarrow$ Circumcentre (C_1) of OAB = $\left(\frac{1+1}{2}, \frac{1-1}{2}\right)$

$\Rightarrow C_1 = (1,0)$

The radius of the circle is half of the length of the hypotenuse.

$\Rightarrow r_1 = \frac{2}{2}$

$\Rightarrow r_1 = 1$ units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation to get the circle’s equation for ΔOAB:

$\Rightarrow (x-1)^2 + (y-0)^2 = (1)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 = 1$

$\Rightarrow x^2 + y^2 - 2x = 0$

$\Rightarrow$ Circumcentre (C_2) of OAD = $\left(\frac{1-1}{2}, \frac{1+1}{2}\right)$

$\Rightarrow C_2 = (0,1)$

The radius of the circle is half of the length of the hypotenuse.

$\Rightarrow r_2 = \frac{2}{2}$

$\Rightarrow r_2 = 1$ units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation to get the circle’s equation for ΔOAD:

$\Rightarrow (x-0)^2 + (y-1)^2 = (1)^2$

$\Rightarrow x^2 + y^2 - 2y + 1 = 1$

$\Rightarrow x^2 + y^2 - 2y = 0$

$\Rightarrow$ Circumcentre (C_3) of OBC = $\left(\frac{-1+1}{2}, \frac{-1-1}{2}\right)$

$\Rightarrow C_3 = (0, - 1)$

The radius of the circle is half of the length of the hypotenuse.

$\Rightarrow r_3 = \frac{2}{2}$

$\Rightarrow r_3 = 1$ units.

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation to get the circle’s equation for ΔOBC:

$\Rightarrow (x-0)^2 + (y-(-1))^2 = (1)^2$

$\Rightarrow x^2 + y^2 + 2y + 1 = 1$

$\Rightarrow x^2 + y^2 + 2y = 0$

$\Rightarrow$ Circumcentre (C_4) of OCD = $\left(\frac{-1-1}{2}, \frac{-1+1}{2}\right)$

$\Rightarrow C_4 = (-1,0)$

The radius of the circle is half of the length of the hypotenuse.

$\Rightarrow r_4 = \frac{2}{2}$

$\Rightarrow r_4 = 1$ units.

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation to get the circle's equation for ΔOAC:

$\Rightarrow (x - (-1))^2 + (y - 0)^2 = (1)^2$

$\Rightarrow x^2 + 2x + 1 + y^2 = 1$

$\Rightarrow x^2 + y^2 + 2x = 0$

∴The equation of the circles are $x^2 + y^2 - 2x = 0$, $x^2 + y^2 - 2y = 0$, $x^2 + y^2 + 2y = 0$ and $x^2 + y^2 + 2x = 0$.

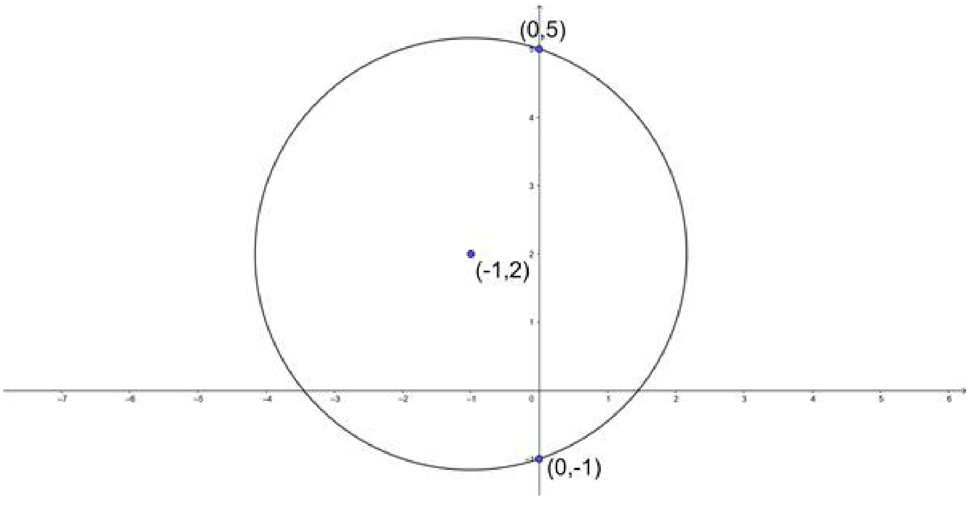
Very Short Answer

1. Question

Write the length of the intercept made by the circle $x^2 + y^2 + 2x – 4y – 5 = 0$ on y - axis.

Answer

Given that we need to find the length of the intercept made by the circle $x^2 + y^2 + 2x - 4y - 5 = 0$ on the y - axis.



For finding the length of intercept we first need to find the intercept made by the circle with the y - axis.

We know that on the y - axis, the value of x is 0.

By substituting x = 0 in the circle equation we get,

$\Rightarrow 0^2 + y^2 + 2(0) - 4y - 5 = 0$

$\Rightarrow y^2 - 4y - 5 = 0$

$\Rightarrow y^2 - 5y + y - 5 = 0$

$\Rightarrow y(y - 5) + 1(y - 5) = 0$

$\Rightarrow (y + 1)(y - 5) = 0$

$\Rightarrow y + 1 = 0$ or $y - 5 = 0$

$\Rightarrow y = -1$ or $y = 5$.

The points intersected by the circle are A(- 1,0) and B(5,0).

We know the length of the intercept is AB.

$\Rightarrow AB = \sqrt{(-1-5)^2 + (0-0)^2}$

$\Rightarrow AB = \sqrt{(-6)^2}$

⇒ AB = 6.

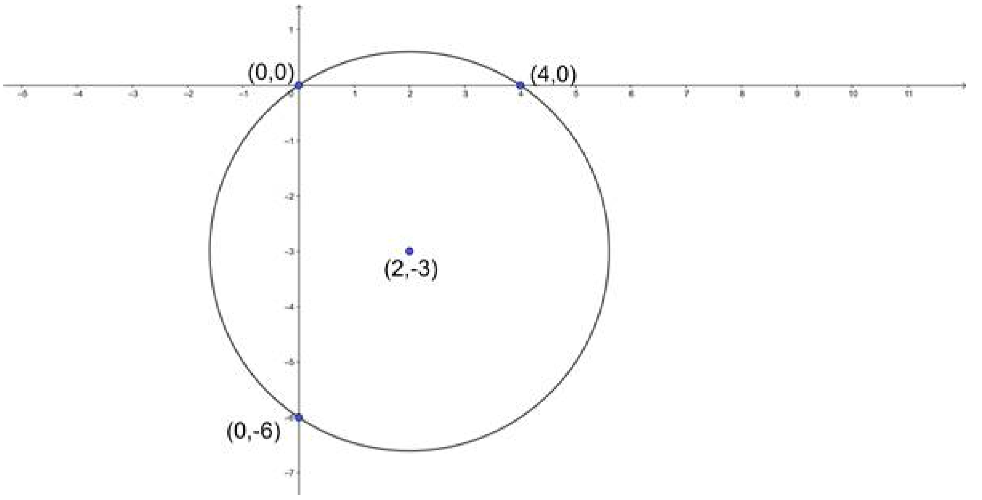
∴The length of the intercept is 6.

2. Question

Write the coordinates of the centre of the circle passing through (0, 0), (4, 0) and (0, - 6).

Answer

Given that we need to find the centre of the circle passing through O(0,0), A(4,0) and B(0, - 6).



Let us find the lengths of the triangle OAB.

We know that the distance between the points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

$\Rightarrow OA = \sqrt{(0-4)^2 + (0-0)^2}$

$\Rightarrow OA = \sqrt{(-4)^2 + 0^2}$

⇒ OA = 4

$\Rightarrow OB = \sqrt{(0-0)^2 + \left(0-(-6)\right)^2}$

$\Rightarrow OB = \sqrt{0^2 + (6)^2}$

⇒ OB = 6

$\Rightarrow AB = \sqrt{(4-0)^2 + \left(0-(-6)\right)^2}$

$\Rightarrow AB = \sqrt{4^2 + 6^2}$

⇒ AB = √52

Consider $OA^2 + OB^2$,

$\Rightarrow OA^2 + OB^2 = 4^2 + 6^2$

$\Rightarrow OA^2 + OB^2 = 16 + 36$

$\Rightarrow OA^2 + OB^2 = 52$

$\Rightarrow OA^2 + OB^2 = (\sqrt{52})^2$

$\Rightarrow OA^2 + OB^2 = AB^2$

We got triangle OAB is a right-angled triangle with AB as the hypotenuse.

We know that the circumcentre of the right-angled triangle is the midpoint of the hypotenuse.

$\Rightarrow$ Centre = $\left(\frac{4+0}{2}, \frac{0-6}{2}\right)$

$\Rightarrow$ Centre = (2, - 3)

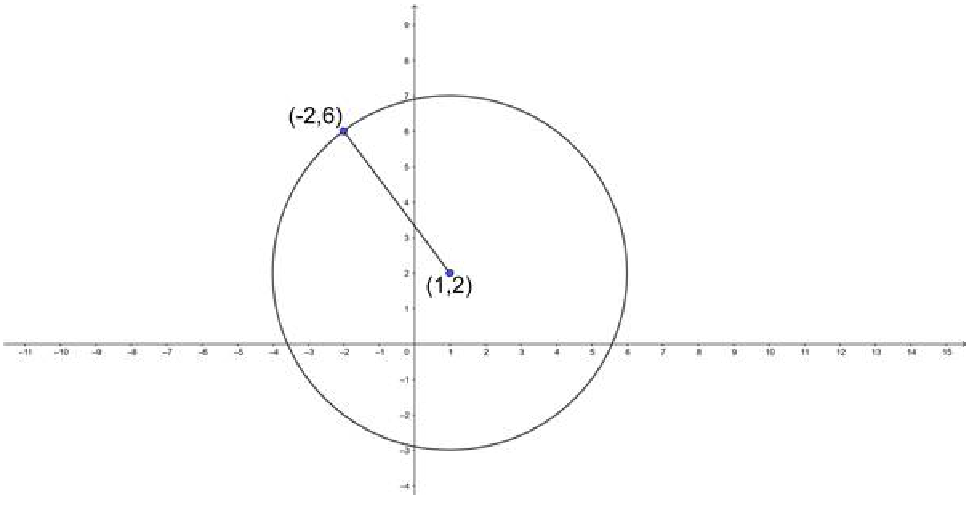
∴The coordinates of the centre is (2, - 3).

3. Question

Write the area of the circle passing through (- 2, 6) and having its centre at (1, 2).

Answer

We need to find the area of the circle passing through (- 2,6) and having a centre at (1,2).



We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(1-(-2))^2 + (2-6)^2}$

$\Rightarrow r = \sqrt{(3)^2 + (-4)^2}$

$\Rightarrow r = \sqrt{9 + 16}$

$\Rightarrow r = \sqrt{25}$

$\Rightarrow r = 5$

We know that the area of the circle is πr^2.

$\Rightarrow$ Area(A) = $\pi(5)^2$

$\Rightarrow A = \pi(25)$

∴The area of the circle is 25π.

4. Question

If the abscissae and ordinates of two points P and Q are roots of the equations $x^2 + 2ax - b^2 = 0$ and $x^2 + 2px - q^2 = 0$ respectively, then write the equation of the circle with PQ as diameter.

Answer

Given that points, we need to find the equation of the circle whose ends of the diameter are P and Q.

It is also told that the abscissae of two points P and Q are the roots of $x^2 + 2ax - b^2 = 0$ and ordinates are the roots of $x^2 + 2px - q^2 = 0$.

Let us first find the roots of each quadratic equation.

For $x^2 + 2ax - b^2 = 0$

$$\Rightarrow x = \frac{-2a \pm \sqrt{(2a)^2 - 4(1)(-b^2)}}{2(1)}$$

$$\Rightarrow x = \frac{-2a \pm \sqrt{4(a^2 + b^2)}}{2}$$

$$\Rightarrow x = -a \pm \sqrt{a^2 + b^2} \text{ ..... (1)}$$

For $x^2 + 2px - q^2 = 0$

$$\Rightarrow x = \frac{-2p \pm \sqrt{(2p)^2 - 4(1)(-q^2)}}{2(1)}$$

$$\Rightarrow x = \frac{-2p \pm \sqrt{4(p^2 + q^2)}}{2}$$

$$\Rightarrow x = -p \pm \sqrt{p^2 + q^2} \text{ ..... (2)}$$

From (1) and (2) we get,

$$\Rightarrow P = \left(-a + \sqrt{a^2 + b^2}, -p + \sqrt{p^2 + q^2}\right)$$

$$\Rightarrow Q = \left(-a - \sqrt{a^2 + b^2}, -p - \sqrt{p^2 + q^2}\right)$$

We know that the centre is the mid - point of the diameter.

$$\Rightarrow \text{Centre(C)} = \left(\frac{-a + \sqrt{a^2 + b^2} - a - \sqrt{a^2 + b^2}}{2}, \frac{-p + \sqrt{p^2 + q^2} - p - \sqrt{p^2 + q^2}}{2}\right)$$

$\Rightarrow$ C = (- a, - p)

We have a circle with centre (- a, - p) and passing through the point $\left(-a - \sqrt{a^2 + b^2}, -p - \sqrt{p^2 + q^2}\right)$.

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$$\Rightarrow r = \sqrt{\left(-a - \left(-a - \sqrt{a^2 + b^2}\right)\right)^2 + \left(-p - \left(-p - \sqrt{p^2 + q^2}\right)\right)^2}$$

$$\Rightarrow r = \sqrt{\left(\sqrt{a^2 + b^2}\right)^2 + \left(\sqrt{p^2 + q^2}\right)^2}$$

$$\Rightarrow r = \sqrt{a^2 + b^2 + p^2 + q^2}$$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$$\Rightarrow (x - (-a))^2 + (y - (-p))^2 = \left(\sqrt{a^2 + b^2 + p^2 + q^2}\right)^2$$

⇒ $x^2 + 2ax + a^2 + y^2 + 2py + p^2 = a^2 + b^2 + p^2 + q^2$

⇒ $x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

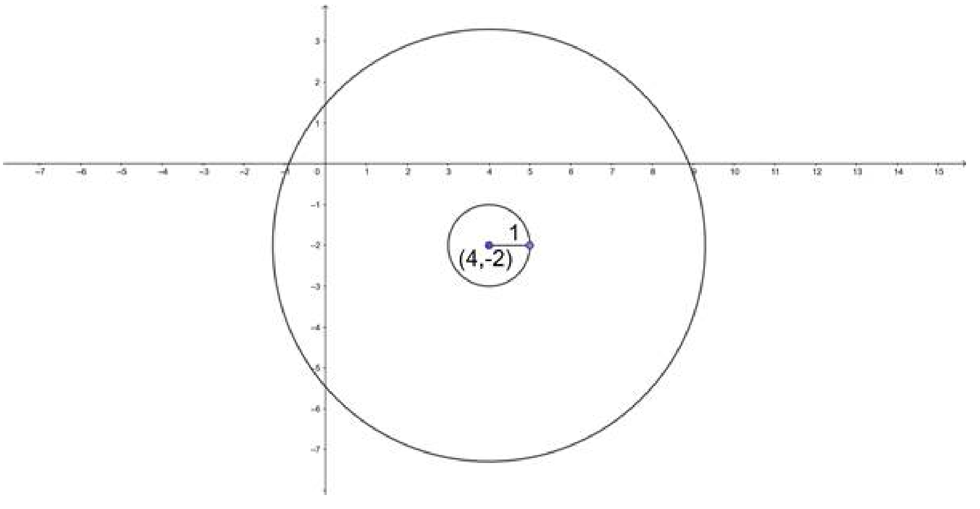
∴The equation of the circle is $x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$.

5. Question

Write the equation of the unit circle concentric with $x^2 + y^2 – 8x + 4y – 8 = 0$.

Answer

Given that we need to find the equation of the circle which is concentric with $x^2 + y^2 - 8x + 4y - 8 = 0$ and having a unit radius.



We know that concentric circles will have the same centre.

Let us assume the concentric circle be $x^2 + y^2 - 8x + 4y + c = 0$. ...... (ii)

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$ ...... (1)

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

We know that unit circle has radius 1.

⇒ $1 = \sqrt{(-4)^2 + 2^2 - c}$

⇒ 1 = 16 + 4 - c

⇒ c = 19

∴ The equation of the concentric circle is $x^2 + y^2 - 8x + 4y + 19 = 0$.

6. Question

If the radius of the circle $x^2 + y^2 + ax + (1 – a) y + 5 = 0$ does not exceed 5, write the number of integral values a.

Answer

Given equation of circle is $x^2 + y^2 + ax + (1 - a)y + 5 = 0$.

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

We need to find the values of ‘a’ such that the radius of the given circle does not exceed 5.

We know that the radius of a circle cannot be less than 0.

Let ‘r’ be the radius of the given circle.

$\Rightarrow 0 \le r \le 5$

$\Rightarrow 0 \le \sqrt{\left(\frac{-a}{2}\right)^2 + \left(\frac{a-1}{2}\right)^2 - 5} \le 5$

$\Rightarrow 0 \le \frac{a^2}{4} + \frac{a^2-2a+1}{4} - 5 \le 25$

$\Rightarrow 0 \le 2a^2 - 2a - 19 \le 100$

By trial and error method we get the set of values of ‘a’ as [- 7.2, 8.2].

∴The integral values of ‘a’ are - 7, - 6, - 5, - 4, - 3, - 2, - 1,0,1,2,3,4,5,6,7,8.

7. Question

Write the equation of the circle passing through (3, 4) and touching y-axis at the origin.

Answer

Given: Circle passes through (3, 4) and touch y-axis at origin, this means it also passes though origin O(0, 0).

To Find: Equation of Circle

General equation of Circle: $x^2 + y^2 + 2gx + 2fy + c = 0$

As circle passes through (0, 0). This point will satisfy the equation.

Therefore,

$(0)^2 + (0)^2 + 2g(0) + 2f(0) + c = 0$

$c = 0$

Now, we have the equation of circle as,

$\mathbf{x^2 + y^2 + 2gx + 2fy = 0}$

Now, since the centre is on the x-axis, y coordinate of centre = 0. i.e. f = 0.

Therefore,

$\mathbf{x^2 + y^2 + 2gx = 0}$

It is also given that this circle passes through (3, 4). So,

$(3)^2 + (4)^2 + 2g(3) = 0$

$9 + 16 + 6g = 0$

$25 + 6g = 0$

$6g = -25$

$$g = -\frac{25}{6}$$

Hence, we have the equation as,

$$\mathbf{x^2 + y^2 - \frac{25}{3}x = 0}$$

8. Question

If the line y = mx does not intersect the circle $(x + 10)^2 + (y + 10)^2 = 180$, then write the set of values taken by m.

Answer

Given that we find the values of 'm' such that y = mx does not intersect the circle $(x + 10)^2 + (y + 10)^2 = 180$

The line does not intersect the circle if the perpendicular distance between the centre and the line is greater than the radius of the circle.

Here the centre and radius of the circle is (- 10, - 10) and √180.

We know that the perpendicular distance from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

$\Rightarrow \frac{|10-m(10)|}{\sqrt{1 + m^2}} > \sqrt{180}$

$\Rightarrow |10 - 10m| > \sqrt{180} \times \sqrt{1 + m^2}$

$\Rightarrow (|10 - 10m|)^2 > \left(\sqrt{180} \times \sqrt{1 + m^2}\right)^2$

$\Rightarrow 100 - 200m + 100m^2 > 180 + 180m^2$

$\Rightarrow 80m^2 + 200m + 80 < 0$

$\Rightarrow 2m^2 + 5m + 2 < 0$

$\Rightarrow m^2 + \frac{5}{2}m + 1 < 0$

$\Rightarrow m^2 + \frac{1}{2}m + 2m + 1 < 0$

$\Rightarrow m\left(m + \frac{1}{2}\right) + 2\left(m + \frac{1}{2}\right) < 0$

$\Rightarrow (m + 2)\left(m + \frac{1}{2}\right) < 0$

We know that the solution set for the inequality (x - a)(x - b)<0 is (a, b) for b>a.

$\Rightarrow m \epsilon \left(-2, \frac{-1}{2}\right)$

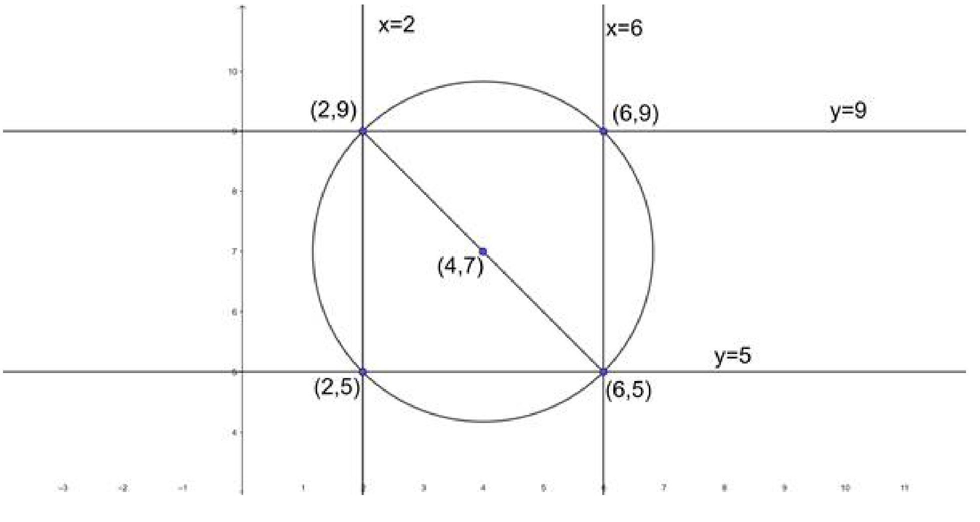
∴The solution set for 'm' is $\left(-2, \frac{-1}{2}\right)$.

9. Question

Write the coordinates of the centre of the circle inscribed in the square formed by the lines x = 2, x = 6, y = 5 and y = 9.

Answer

Given that we need to find the centre of the circle inscribed square.



It is also told that x = 2, x = 6, y = 5 and y = 9 are the sides of a square.

Let us assume A,B,C,D are the vertices of the square. On solving the lines we get the vertices as:

⇒ A = (2,5)

⇒ B = (6,5)

⇒ C = (6,9)

⇒ D = (2,9)

Since the diagonal of the square is diameter of circle as the circle circumscribes the square.

So, taking any diagonal as diameter gives the same centre of the circle.

Let us assume diagonal AC as the diameter.

We know that centre is the mid - point of the diameter.

⇒ Centre(C) = $\left(\frac{2+6}{2}, \frac{5+9}{2}\right)$

⇒ C = (4,7)

∴The coordinates of the centre is (4,7).

MCQ

1. Question

If the equation of a circle is $\lambda x^2 + (2\lambda - 3)y^2 - 4x + 6y - 1 = 0$, then the coordinates of centre are

A. $\left(\frac{4}{3}, -1\right)$

B. $\left(\frac{2}{3}, -1\right)$

C. $\left(\frac{-2}{3}, -1\right)$

D. $\left(\frac{2}{3}, 1\right)$

Answer

Given that the equation of the circle is:

⇒ $\lambda x^2 + (2\lambda - 3)y^2 - 4x + 6y - 1 = 0$ ..... (1)

Comparing with the standard equation of circle:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$

We get

⇒ $\lambda = 2\lambda - 3$

⇒ $2\lambda - \lambda = 3$

⇒ $\lambda = 3$

Substituting λ value in (1), we get

⇒ $3x^2 + (2(3) - 3)y^2 - 4x + 6y - 1 = 0$

⇒ $3x^2 + 3y^2 - 4x + 6y - 1 = 0$

$\Rightarrow x^2 + y^2 - \frac{4}{3}x + 2y - \frac{1}{3} = 0$

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

⇒ Centre(C) = $\left(\frac{-\left(\frac{-4}{3}\right)}{2}, \frac{-2}{2}\right)$

$\Rightarrow C = \left(\frac{2}{3}, -1\right)$

∴The correct option is (b)

2. Question

If $2x^2 + \lambda xy + 2y^2 + (\lambda - 4)x + 6y - 5 = 0$ is the equation of a circle, then its radius is

A. $3\sqrt{2}$

B. $2\sqrt{3}$

C. $2\sqrt{2}$

D. none of the above

Answer

Given that the equation of the circle is:

⇒ $2x^2 + \lambda xy + 2y^2 + (\lambda - 4)x + 6y - 5 = 0$ ..... (1)

Comparing with the standard equation of circle:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$

We get

⇒ $\lambda = 0$

Substituting λ value in (1), we get

⇒ $2x^2 + 0xy + 2y^2 + (0 - 4)x + 6y - 5 = 0$

⇒ $2x^2 + 2y^2 - 4x + 6y - 5 = 0$

$\Rightarrow x^2 + y^2 - 2x + 3y - \frac{5}{2} = 0$

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

⇒ Radius (r) = $\sqrt{(-1)^2 + \left(\frac{3}{2}\right)^2 - \left(\frac{-5}{2}\right)}$

$\Rightarrow r = \sqrt{1 + \frac{9}{4} + \frac{5}{2}}$

$\Rightarrow r = \sqrt{\frac{23}{4}}$

$\Rightarrow r = \frac{\sqrt{23}}{2}$

∴The correct option is (d)

3. Question

Mark the correct alternatives in each of the following :

The equation $x^2 + y^2 + 2x – 4y + 5 = 0$ represents

A. A point

B. A pair of straight lines

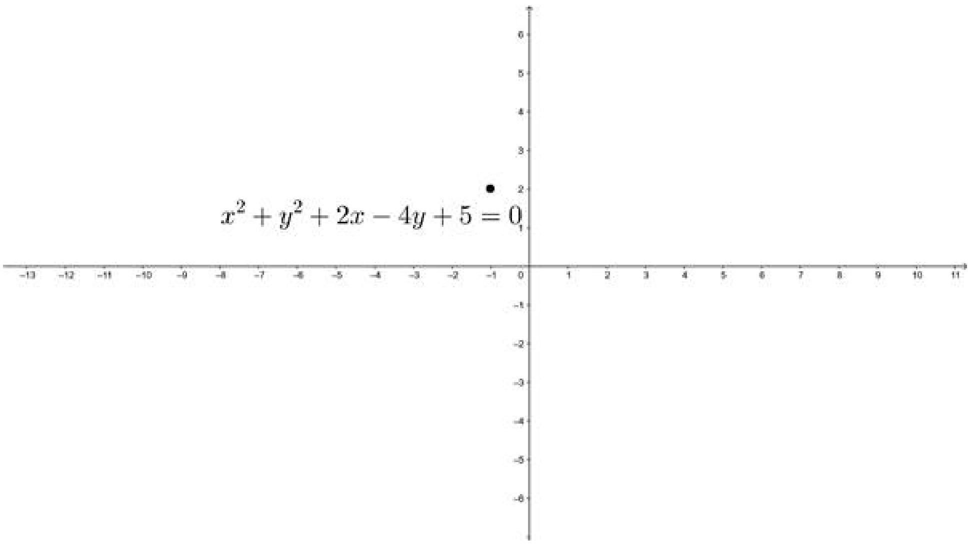
C. A circle of non - zero radius

D. None of these

Answer

Given equation is:

⇒ $x^2 + y^2 + 2x - 4y + 5 = 0$



We know that for a pair of straight lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, the conditions to be satisfied is $abc + 2fgh - af^2 - bg^2 - cf^2 = 0$, $h^2 \geq ab$, $g^2 \geq ca$ and $f^2 \geq bc$.

Here we can see that h = 0 a = b = 1

⇒ $h^2 = 0^2 = 0$

⇒ ab = 1.1 = 1

⇒ $h^2 \leq ab$. So, the equation doesn’t represent a pair of straight lines.

⇒ $x^2 + y^2 + 2x - 4y + 5 = 0$

⇒ $(x^2 + 2x + 1) + (y^2 - 4y + 4) = 0$

⇒ $(x + 1)^2 + (y - 2)^2 = 0$

The equation represents circle with zero radius. This is a point circle.

∴The correct option is (a).

4. Question

If the equation $(4a – 3)\ x^2 + ay^2 + 6x – 2y + 2 = 0$ represents a circle, then its centre is

A. (3, - 1)

B. (3,1)

C. (- 3,1)

D. none of these

Answer

Given that the equation of the circle is:

⇒ $(4a - 3)x^2 + ay^2 + 6x - 2y + 2 = 0$ ..... (1)

Comparing with the standard equation of circle:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$

We get

⇒ $4a - 3 = a$

⇒ $4a - a = 3$

⇒ $a = 1$

Substituting a value in (1), we get

⇒ $(4(1) - 3)x^2 + 1y^2 + 6x - 2y + 2 = 0$

⇒ $x^2 + y^2 + 6x - 2y + 2 = 0$

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

⇒ Centre(C) = $\left(\frac{-(6)}{2}, \frac{-(-2)}{2}\right)$

⇒ $C = (-3,1)$

∴The correct option is (c)

5. Question

The radius of the circle represented by equation $3x^2 + 3y^2 + \lambda xy + 9x + (\lambda - 6)y + 3 = 0$ is

A. $\frac{3}{2}$

B. $\frac{\sqrt{17}}{2}$

C. $\frac{2}{3}$

D. none of these

Answer

Given that the equation of the circle is:

⇒ $3x^2 + 3y^2 + \lambda xy + 9x + (\lambda - 6)y + 3 = 0$ ...... (1)

Comparing with the standard equation of circle:

⇒ $x^2 + y^2 + 2ax + 2by + c = 0$

We get

⇒ $\lambda = 0$

Substituting λ value in (1), we get

$\Rightarrow 3x^2 + 3y^2 + 0xy + 9x + (0 - 6)y + 3 = 0$

$\Rightarrow 3x^2 + 3y^2 + 9x - 6y + 3 = 0$

$\Rightarrow x^2 + y^2 + 3x - 2y + 1 = 0$

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

$\Rightarrow$ Centre = (- a, - b)

$\Rightarrow$ Radius = $\sqrt{a^2 + b^2 - c}$

$\Rightarrow$ Radius (r) = $\sqrt{\left(\frac{-3}{2}\right)^2 + (-1)^2 - (1)}$

$\Rightarrow r = \sqrt{\frac{9}{4} + 1 - 1}$

$\Rightarrow r = \sqrt{\frac{9}{4}}$

$\Rightarrow r = \frac{3}{2}$

∴The correct option is (a)

6. Question

The number of integral values of λ for which the equation $x^2 + y^2 + \lambda x + (1 - \lambda) y + 5 = 0$ is the equation of a circle whose radius cannot exceed 5, is

A. 14

B. 18

C. 16

D. none of these

Answer

Given equation of circle is $x^2 + y^2 + \lambda x + (1 - \lambda)y + 5 = 0$.

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

$\Rightarrow$ Centre = (- a, - b)

$\Rightarrow$ Radius = $\sqrt{a^2 + b^2 - c}$

We need to find the values of 'a' such that the radius of the given circle does not exceed 5.

We know that the radius of a circle cannot be less than 0.

Let 'r' be the radius of the given circle.

$\Rightarrow 0 \le r \le 5$

$\Rightarrow 0 \le \sqrt{\left(\frac{-\lambda}{2}\right)^2 + \left(\frac{\lambda-1}{2}\right)^2 - 5} \le 5$

$\Rightarrow 0 \le \frac{\lambda^2}{4} + \frac{\lambda^2 - 2\lambda + 1}{4} - 5 \le 25$

$\Rightarrow 0 \le 2\lambda^2 - 2\lambda - 19 \le 100$

By trial and error method we get the set of values of 'λ' as [- 7.2, 8.2].

The integral values of 'λ' are - 7, - 6, - 5, - 4, - 3, - 2, - 1,0,1,2,3,4,5 ,6,7,8.

The no. of values of values of 'λ' is 16.

∴The correct option is (c)

7. Question

The equation of the circle passing through the point (1, 1) and having two diameters along the pair of lines $x^2 - y^2 - 2x - 3 = 0$, is

A. $x^2 + y^2 - 2x - 4y + 4 = 0$

B. $x^2 + y^2 + 2x + 4y - 4 = 0$

C. $x^2 + y^2 - 2x + 4y + 4 = 0$

D. none of these

Answer

Given that we need to find the equation of the circle passing through the point (1,1) and having two diameters along the pair of lines $x^2 - y^2 - 2x - 3 = 0$.

We know that the point of intersection of the pair of straight lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$.

Let us assume the intersection point of the pair of lines be 'O'.

$\Rightarrow O = \left(\frac{((0)(0))-((-1)(-1))}{((1)(-1))-(0)^2}, \frac{((-2)(0))-((1)(0))}{((1)(-1))-(0)^2}\right)$

$\Rightarrow O = \left(\frac{-1}{-1}, \frac{0}{-1}\right)$

$\Rightarrow O = (1,0)$

We have a circle with centre (1,0) and passing through the point (1,1).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(1-1)^2 + (0-1)^2}$

$\Rightarrow r = \sqrt{(0)^2 + (-1)^2}$

$\Rightarrow r = \sqrt{0 + 1}$

$\Rightarrow r = 1$

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x - (1))^2 + (y - 0)^2 = (1)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 = 1$

$\Rightarrow x^2 + y^2 - 2x = 0$

∴The correct option is (d).

8. Question

If the centroid of an equilateral triangle is (1, 1) and its one vertex is (- 1, 2), then the equation of its circumcircle is

A. $x^2 + y^2 - 2x - 2y - 3 = 0$

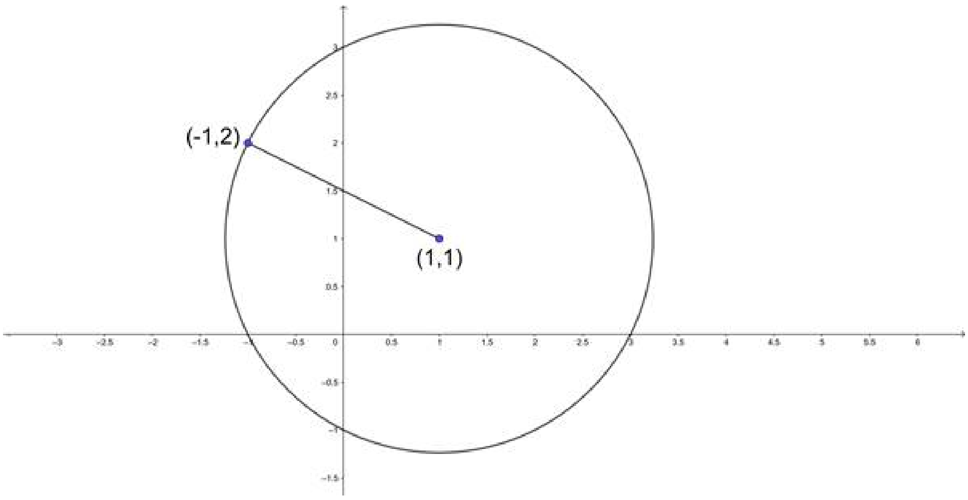
B. $x^2 + y^2 + 2x - 2y - 3 = 0$

C. $x^2 + y^2 + 2x + 2y - 3 = 0$

D. none of the these

Answer

Given that we need to find the equation of circumcircle of an equilateral triangle whose centroid is (1,1) and one of its vertex is (- 1,2).



We know that in an equilateral triangle, the centroid and circumcentre coincides and circumcircle passes through all the vertices of the triangle.

We have a circle with centre (1,1) and passing through the point (- 1,2).

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

We know that the distance between the two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Let us assume r is the radius of the circle.

$\Rightarrow r = \sqrt{(1-(-1))^2 + (1-2)^2}$

$\Rightarrow r = \sqrt{(2)^2 + (-1)^2}$

$\Rightarrow r = \sqrt{4 + 1}$

$\Rightarrow r = \sqrt{5}$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x-1)^2 + (y-1)^2 = (\sqrt{5})^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 2y + 1 = 5$

$\Rightarrow x^2 + y^2 - 2x - 2y - 3 = 0$

∴The correct option is (a).

9. Question

If the point (2, k) lies outside the circles $x^2 + y^2 + x - 2y - 14 = 0$ and $x^2 + y^2 = 13$ then k lies in the interval

A. (- 3, - 2) ∪ (3, 4)

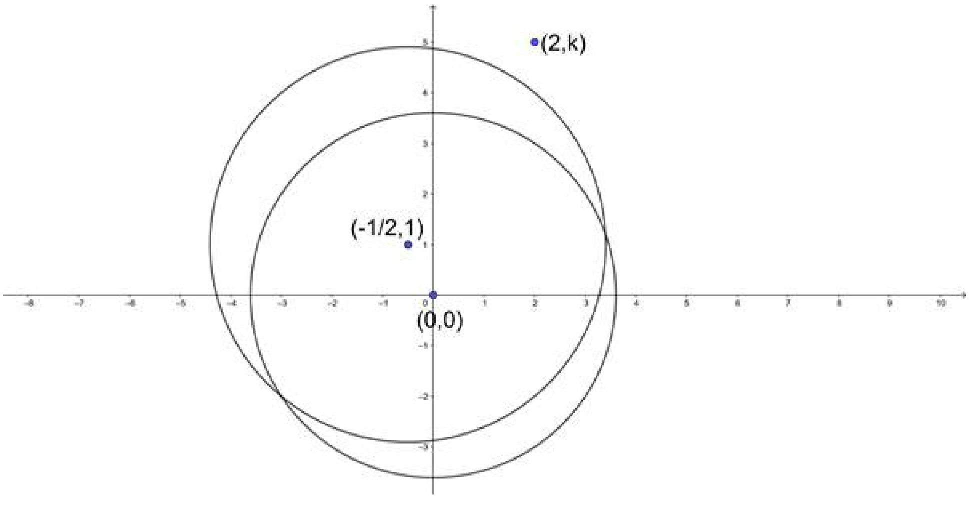
B. - 3, 4

C. (- ∞, - 3) ∪ (4, ∞)

D. (- ∞, - 2) ∪ (3, ∞)

Answer

Given that point (2,k) lies outsides the circles $x^2 + y^2 + x - 2y - 14 = 0$ and $x^2 + y^2 = 13$.



We know that for a point (a, b) to lie outside the circle S, the condition to be satisfies is $S_{11}>0$.

Applying $S_{11}>0$ for 1st circle,

$\Rightarrow 2^2 + k^2 + 2 - 2(k) - 14>0$

$\Rightarrow 4 + k^2 - 2k - 12>0$

$\Rightarrow k^2 - 2k - 8>0$

$\Rightarrow k^2 - 4k + 2k - 8>0$

$\Rightarrow k(k - 4) + 2(k - 4)>0$

$\Rightarrow (k + 2)(k - 4)>0$

We know that the solution set of (x - a)(x - b)>0 for b>a is $(-\infty,a)\cup(b,\infty)$.

The solution set for k is $(-\infty, -2)\cup(4,\infty)$ ..... (1)

Applying $S_{11}>0$ for 2nd circle,

$\Rightarrow 2^2 + k^2 - 13>0$

$\Rightarrow k^2 + 4 - 13>0$

$\Rightarrow k^2 - 9>0$

$\Rightarrow (k - 3)(k + 3)>0$

The solution set for k is $(-\infty, -3)\cup(3,\infty)$ ...... (2)

The resultant solution set for k is the intersection of (1) and (2).

$\Rightarrow k\in(((-\infty, -2)\cup(4,\infty))\cap((-\infty, -3)\cup(3,\infty)))$

$\Rightarrow k\in((-\infty, -3)\cup(4,\infty))$

∴The correct option is (c).

10. Question

If the point (λ, λ + 1) lies inside the region bounded by the curve $x = \sqrt{25 - y^2}$ and y - axis, then λ belongs to the interval

A. (- 1,3)

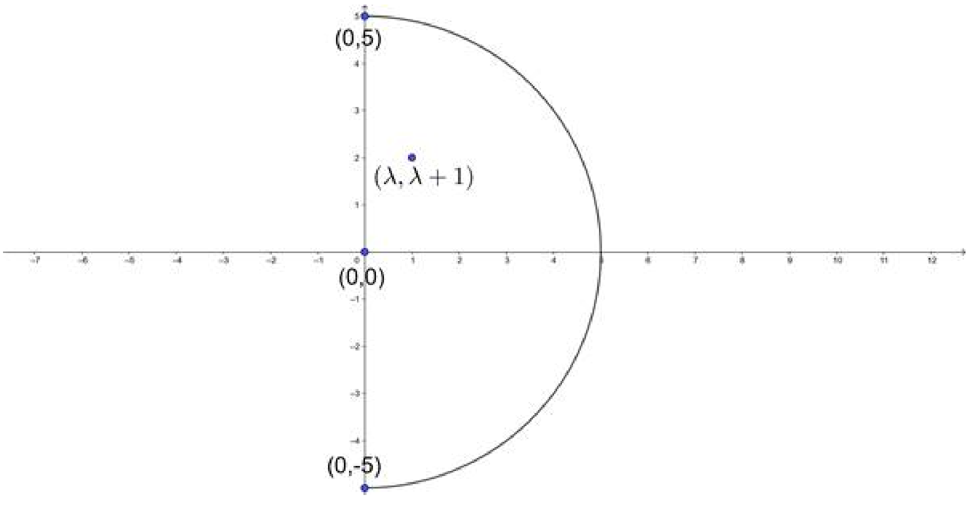
B. (- 4,2)

C. (- ∞, - 4)∪(3,∞)

D. none of these

Answer

Given that the point (λ, λ + 1) lies inside the region bounded by the curve $x = \sqrt{25 - y^2}$ and y - axis.



The curve is rewritten as,

$\Rightarrow x^2 = 25 - y^2$

$\Rightarrow x^2 + y^2 = 25$

$\Rightarrow S = x^2 + y^2 - 25$

We know that for a point (a, b) to lie inside the circle S, the condition to be satisfied is $S_{11} < 0$.

Applying $S_{11} < 0$ for 1st circle,

$\Rightarrow \lambda^2 + (\lambda + 1)^2 - 25 < 0$

$\Rightarrow \lambda^2 + \lambda^2 + 2\lambda + 1 - 25 < 0$

$\Rightarrow 2\lambda^2 + 2\lambda - 24 < 0$

$\Rightarrow \lambda^2 + \lambda - 12 < 0$

$\Rightarrow \lambda^2 + 4\lambda - 3\lambda - 12 < 0$

$\Rightarrow \lambda(\lambda + 4) - 3(\lambda + 4) < 0$

$\Rightarrow (\lambda - 3)(\lambda + 4) < 0$

We know that the solution set of (x - a)(x - b)<0 for b>a is (a,b).

The solution set for λ is (- 4,3) ...... (1)

Since the point lies inside y - axis λ + 1>0

$\Rightarrow \lambda > -1$ ..... (2)

The resultant solution set is the intersection of (1) and (2).

$\Rightarrow \lambda \in ((-4,3) \cap (-1,\infty))$

⇒ $\lambda \epsilon (-1,3)$

∴ The correct option is (a).

11. Question

The equation of the incircle formed by the coordinate axes and the line 4x + 3y = 6 is

A. $x^2 + y^2 - 6x - 6y + 9 = 0$

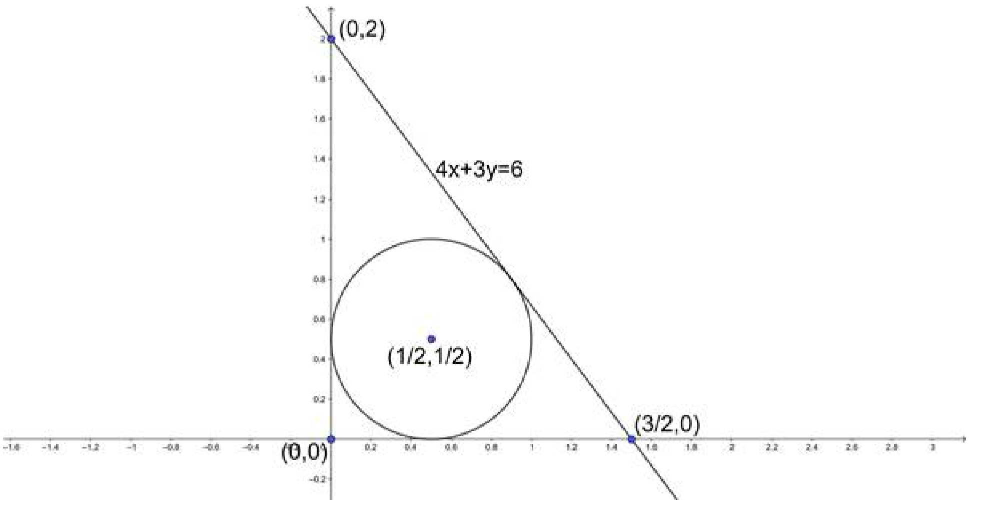
B. $4(x^2 + y^2 - x - y) + 1 = 0$

C. $4(x^2 + y^2 + x + y) + 1 = 0$

D. None of these

Answer

Given that we need to find the equation of the incircle formed by the coordinate axes and the line 4x + 3y = 6.



Let us find the vertices of triangle.

We know that the axes meet at origin O.

When the line meets x - axis, the value of ‘y’ is 0.

⇒ 4x + 3(0) = 6

$\Rightarrow x = \frac{6}{4}$

$\Rightarrow x = \frac{3}{2}$

The point is $A\left(\frac{3}{2}, 0\right)$.

When the line meets y - axis, the value of ‘x’ is 0.

⇒ 4(0) + 3y = 6

$\Rightarrow y = \frac{6}{3}$

⇒ y = 2

The point is B(0,2).

Let us find the length of sides of the triangle.

We know that the distance between two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

$\Rightarrow OA = \sqrt{\left(0 - \frac{3}{2}\right)^2 + (0 - 0)^2}$

$\Rightarrow OA = \sqrt{\left(\frac{3}{2}\right)^2}$

$\Rightarrow OA = a = \frac{3}{2}$

$\Rightarrow OB = \sqrt{(0-0)^2 + (0-2)^2}$

$\Rightarrow OB = \sqrt{(2)^2}$

$\Rightarrow OB = b = 2$

$\Rightarrow AB = \sqrt{\left(\frac{3}{2}-0\right)^2 + (0-2)^2}$

$\Rightarrow AB = \sqrt{\frac{9}{4} + 4}$

$\Rightarrow AB = \sqrt{\frac{25}{4}}$

$\Rightarrow AB = c = \frac{5}{2}$

We know that incentre of a triangle is given by:

$\Rightarrow I = \left(\frac{cx_1 + bx_2 + ax_3}{a+b+c}, \frac{cy_1 + by_2 + ay_3}{a+b+c}\right)$

$\Rightarrow I = \left(\frac{\left(\frac{5}{2}\times0\right) + \left(2\times\frac{3}{2}\right) + \left(\frac{3}{2}\times0\right)}{\frac{5}{2}+2+\frac{3}{2}}, \frac{\left(\frac{5}{2}\times0\right) + (2\times0) + \left(\frac{3}{2}\times2\right)}{\frac{5}{2}+2+\frac{3}{2}}\right)$

$\Rightarrow I = \left(\frac{3}{6}, \frac{3}{6}\right)$

$\Rightarrow I = \left(\frac{1}{2}, \frac{1}{2}\right)$

We have x = 0 as tangent to this circle.

We know that the perpendicular distance between the circle and centre is equal to the radius of the circle.

We know that the perpendicular distance between from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$\Rightarrow r = \frac{\left|\frac{1}{2}\right|}{\sqrt{1^2}}$

$\Rightarrow r = \frac{1}{2}.$

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2$

$\Rightarrow x^2 - x + \frac{1}{4} + y^2 - y + \frac{1}{4} = \frac{1}{4}$

$\Rightarrow 4(x^2 + y^2 - x - y) + 1 = 0$

∴The correct option is (b).

12. Question

If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 8y + c = 0$ touch each other, then c is equal to

A. 15

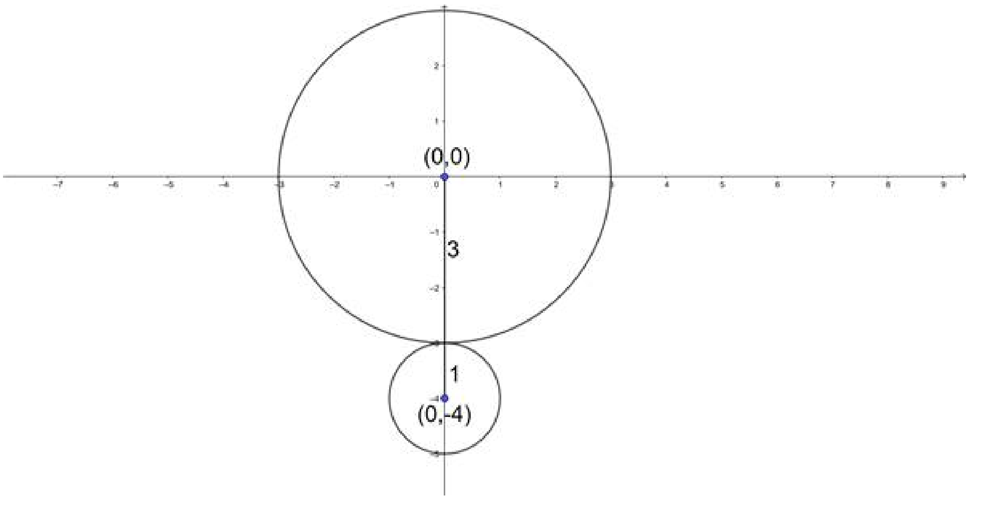
B. - 15

C. 16

D. – 16

Answer

Given that the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 8y + c = 0$ touch each other externally(assumed).



We need to find the value of c.

We know that if the circles touch each other externally, the distance between the centres is equal to the sum of the radii of two circles.

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

$\Rightarrow$ Centre = (- a, - b)

$\Rightarrow$ Radius = $\sqrt{a^2 + b^2 - c}$

For $x^2 + y^2 = 9$

$\Rightarrow$ Centre(C_1) = (0,0)

$\Rightarrow$ Radius(r_1) = $\sqrt{0 + 0 - (-9)}$

$\Rightarrow r_1 = 3$

For $x^2 + y^2 + 8y + c = 0$

$\Rightarrow$ Centre(C_2) = $\left(\frac{0}{2}, \frac{-8}{2}\right)$

$\Rightarrow C_2 = (0, - 4)$

$\Rightarrow$ Radius(r_2) = $\sqrt{0^2 + (-4)^2 - c}$

$\Rightarrow r_2 = \sqrt{16 - c}$

We have $C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{(0-0)^2 + \left(0-(-4)\right)^2} = 3 + \sqrt{16 - c}$

$\Rightarrow 4 = 3 + \sqrt{16 - c}$

$\Rightarrow 1 = \sqrt{16 - c}$

$\Rightarrow 1 = 16 - c$

$\Rightarrow c = 16 - 1$

$\Rightarrow c = 15$

∴The correct option is (a).

13. Question

If the circle $x^2 + y^2 + 2ax + 8y + 16 = 0$ touches x - axis, then the value of a is

A. ±16

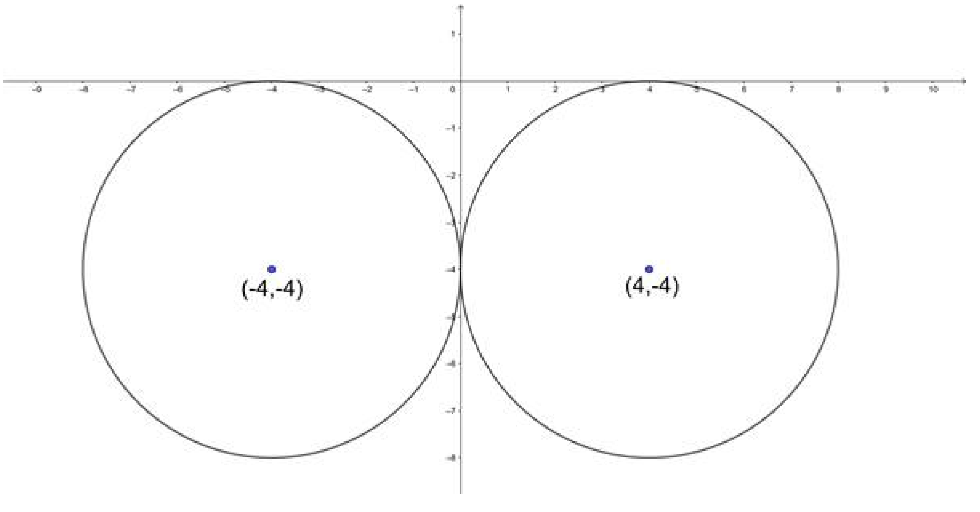
B. ±4

C. ±8

D. ±1

Answer

Given that the circle $x^2 + y^2 + 2ax + 8y + 16 = 0$ touches x - axis. We need to find the value of a.



We know that the value of y on the x - axis is 0.

$\Rightarrow x^2 + (0)^2 + 2ax + 8(0) + 16 = 0$

$\Rightarrow x^2 + 2ax + 16 = 0$

The quadratic equation will have similar roots if the circle touches the x - axis.

We know that for the quadratic equation $ax^2 + bx + c = 0$ the condition to be satisfied for the equal roots is $b^2 - 4ac = 0$

$\Rightarrow (2a)^2 - 4(1)(16) = 0$

$\Rightarrow 4a^2 - 64 = 0$

$\Rightarrow a^2 = 16$

$\Rightarrow a = \pm 4$

∴The correct option is (b).

14. Question

The equation of a circle with radius 5 and touching both the coordinate axes is

A. $x^2 + y^2 \pm 10x \pm 10x + 5 = 0$

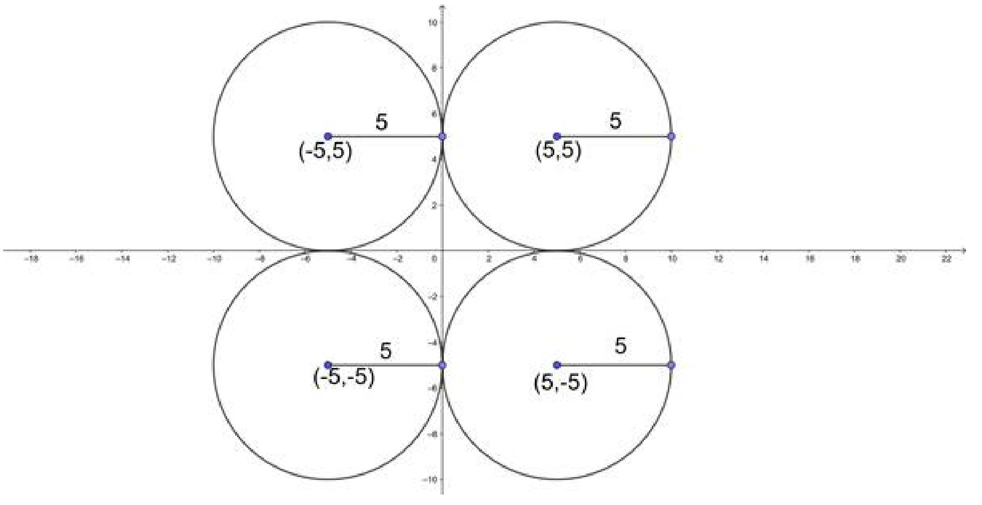
B. $x^2 + y^2 \pm 10x \pm 10y = 0$

C. $x^2 + y^2 \pm 10x \pm 10y + 25 = 0$

D. $x^2 + y^2 \pm 10x \pm 10y + 51 = 0$

Answer

Given that the circle having radius 5 touches both the coordinate axes.



Let us assume the circle touches the co - ordinate axes at (a,0) and (0,a). Then the circle will have the centre at (a, a) and radius |a|.

It is given that the radius is 5 units.

The centre of the circle is (±5,±5).

We know that the equation of the circle with centre (p, q) and having radius ‘r’ is given by:

$\Rightarrow (x - p)^2 + (y - q)^2 = r^2$

Now we substitute the corresponding values in the equation:

$\Rightarrow (x \pm 5)^2 + (y \pm 5)^2 = (5)^2$

$\Rightarrow x^2 \pm 10x + 25 + y^2 \pm 10y + 25 = 25$

$\Rightarrow x^2 + y^2 \pm 10x \pm 10y + 25 = 0$

∴The correct option is (c).

15. Question

The equation of the circle passing through the origin which cuts off intercept of length 6 and 8 from the axes is

A. $x^2 + y^2 - 12x - 16y = 0$

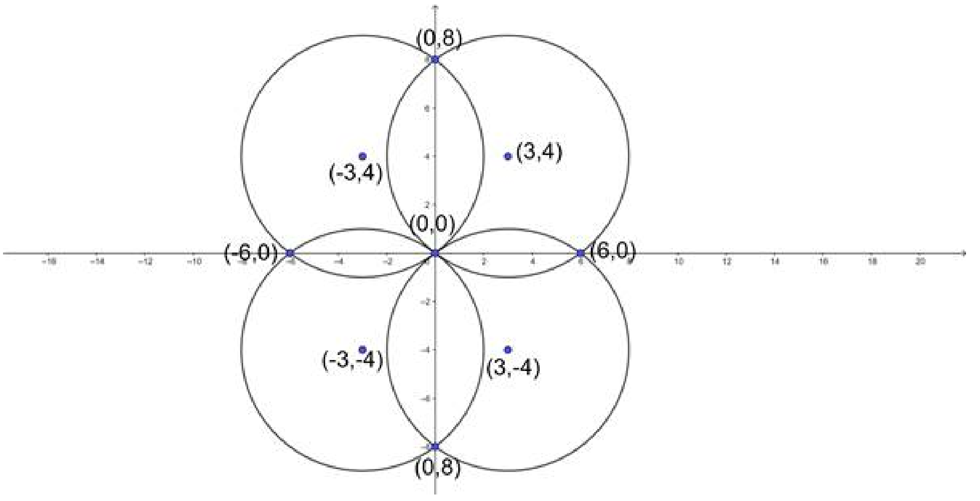
B. $x^2 + y^2 + 12x + 16y = 0$

C. $x^2 + y^2 + 7x + 8y = 0$

D. $x^2 + y^2 - 6x - 8y = 0$

Answer

Given that we need to find the equation of the circle passing through origin and cuts off intercepts 6 and 8 from x and y - axes.



Since the circle is having intercept a from x - axis the circle must pass through (6,0) and (- 6,0) as it already passes through the origin.

Since the circle is having intercept 8 from x - axis the circle must pass through (0,8) and (0, - 8) as it already passes through the origin.

Let us assume the circle passing through the points O(0,0), A(6,0) and B(0,8).

We know that the standard form of the equation of the circle is given by:

$\Rightarrow x^2 + y^2 + 2fx + 2gy + c = 0$ ..... (1)

Substituting O(0,0) in (1), we get,

$\Rightarrow 0^2 + 0^2 + 2f(0) + 2g(0) + c = 0$

$\Rightarrow c = 0$ ..... (2)

Substituting A(6,0) in (1), we get,

$\Rightarrow 6^2 + 0^2 + 2f(6) + 2g(0) + c = 0$

$\Rightarrow 36 + 12f + c = 0$ ..... (3)

Substituting B(0,8) in (1), we get,

$\Rightarrow 0^2 + 8^2 + 2f(0) + 2g(8) + c = 0$

$\Rightarrow 64 + 16g + c = 0$ ..... (4)

On solving (2), (3) and (4) we get,

$\Rightarrow f = -3, b = -4$ and $c = 0$

Substituting these values in (1), we get

$\Rightarrow x + y^2 + 2(-3)x + 2(-4)y + 0 = 0$

$\Rightarrow x^2 + y^2 - 6x - 8y = 0$

Similarly, we get the equation $x^2 + y^2 + 6x + 8y = 0$ for the circle passing through the points (0,0), (- 6,0), (0, - 8), $x^2 + y^2 + 6x - 8y = 0$ for the circle passing through the points (0,0), (- 6,0), (0,8) $x^2 + y^2 - 6x + 8y = 0$ for the circle passing through the points (0,0), (6,0), (0, - 8) .

∴The equations of the circles are $x^2 + y^2 \pm 6x \pm 8y = 0$.

∴The correct options are (d).

16. Question

The equation of the circle concentric with $x^2 + y^2 - 3x + 4y - c = 0$ and passing through (- 1, - 2) is

A. $x^2 + y^2 - 3x + 4y - 1 = 0$

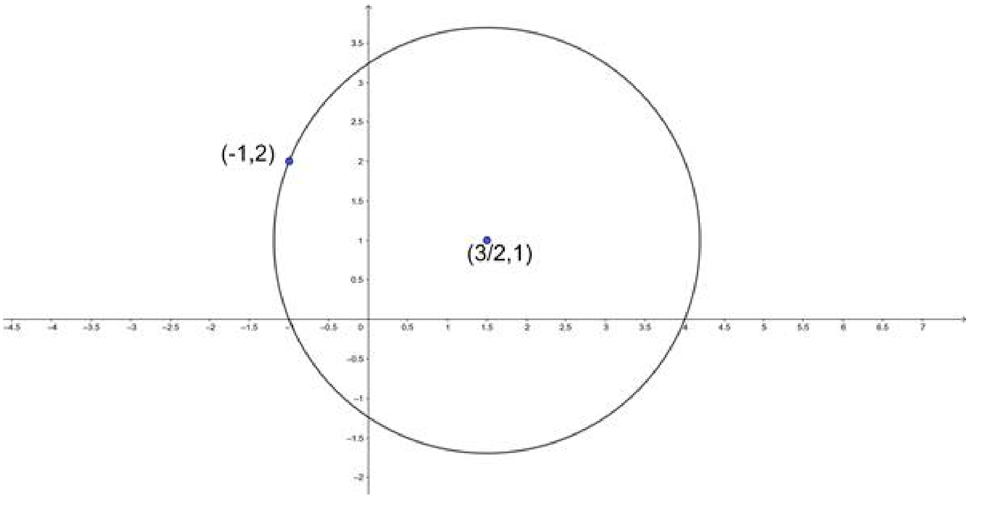
B. $x^2 + y^2 - 3x + 4y = 0$

C. $x^2 + y^2 - 3x + 4y + 2 = 0$

D. none of these

Answer

Given that we need to find the equation of the circle which is concentric with $x^2 + y^2 - 3x + 4y - c = 0$ and passing through (- 1, - 2).



We know that concentric circles will have same centre.

Let us assume the concentric circle be $x^2 + y^2 - 3x + 4y + d = 0$. ..... - (ii)

Substituting (- 1, - 2) in (ii) we get

$\Rightarrow (-1)^2 + (-2)^2 - 3(-1) + 4(-2) + d = 0$

$\Rightarrow 1 + 4 + 3 - 8 + d = 0$

$\Rightarrow d = 0$

Substituting value of d in (ii) we get

$\Rightarrow x^2 + y^2 - 3x + 4y = 0$

∴ The correct option is (b).

17. Question

The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ does not intersect x - axis, if

A. $g^2 < c$

B. $g^2 > c$

C. $g^2 > 2c$

D. none of these

Answer

Given that the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ does not intersect x - axis. We need to find the relation between g and c.

We know that the value of y on the x - axis is 0.

$\Rightarrow x^2 + (0)^2 + 2gx + 2f(0) + c = 0$

$\Rightarrow x^2 + 2gx + c = 0$

The quadratic equation will have no roots if the circle does not intersect the x - axis.

We know that for the quadratic equation $ax^2 + bx + c = 0$ the condition to be satisfied for the equal roots is $b^2 - 4ac<0$

$\Rightarrow (2g)^2 - 4(1)(c)<0$

$\Rightarrow 4(g^2 - c)<0$

$\Rightarrow g^2 - c<0$

$\Rightarrow g^2<c$

∴The correct option is (a).

18. Question

The area of an equilateral triangle inscribed in the circle $x^2 + y^2 - 6x - 8y - 25 = 0$ is

A. $\frac{225\sqrt{3}}{6}$

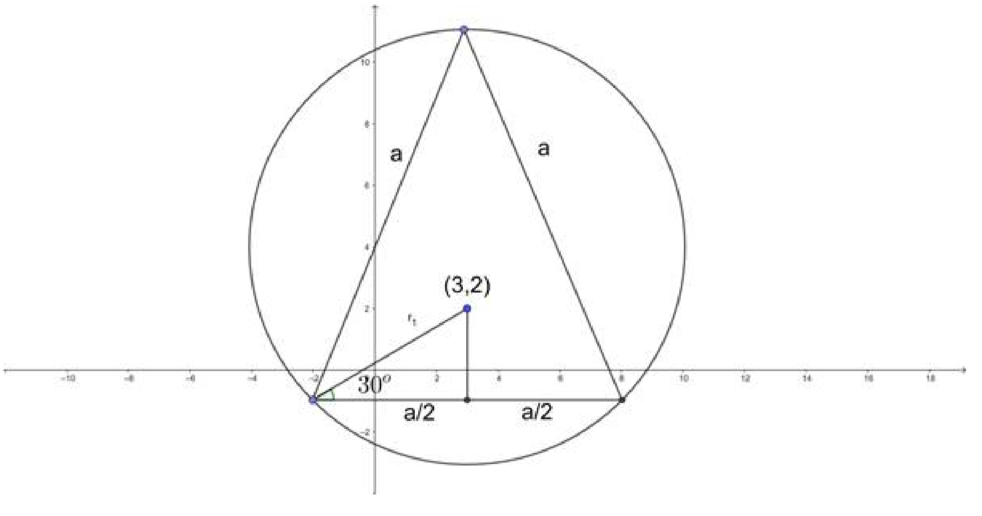
B. 25 π

C. 50 π- 100

D. none of these

Answer

We need to find the area of the equilateral triangle that is inscribed in the circle $x^2 + y^2 - 6x - 8y - 25 = 0$.



We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

$\Rightarrow$ Centre = (- a, - b)

$\Rightarrow$ Radius = $\sqrt{a^2 + b^2 - c}$

For $x^2 + y^2 - 6x - 8y - 25 = 0$

$\Rightarrow$ Radius(r_1) = $\sqrt{3^2 + 4^2 - (-25)}$

$\Rightarrow r_1 = \sqrt{9 + 16 + 25}$

$\Rightarrow r_1 = \sqrt{50}$

$\Rightarrow r_1 = 5\sqrt{2}$

From the figure we can see that,

$\Rightarrow \cos 30^0 = \frac{\frac{a}{2}}{r_1}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a}{2(5\sqrt{2})}$

$\Rightarrow a = 5\sqrt{6}$

We know that area of the equilateral triangle with side length ‘a’ is $\frac{\sqrt{3}}{4}a^2$

$\Rightarrow A = \frac{\sqrt{3}}{4} \times (5\sqrt{6})^2$

$\Rightarrow A = \frac{\sqrt{3}}{4} \times (150)$

$\Rightarrow A = \frac{75\sqrt{3}}{2}$

$\Rightarrow A = \frac{75\sqrt{3}}{2} \times \frac{3}{3}$

$\Rightarrow A = \frac{225\sqrt{3}}{6}$.

∴The correct option is (a).

19. Question

The equation of the circle which touches the axes of coordinates and the line $\frac{x}{3} + \frac{y}{4} = 1$ and whose centres lie in the first quadrant is $x^2 + y^2 - 2cx - 2cy + c^2 = 0$, where c is equal to

A. 4

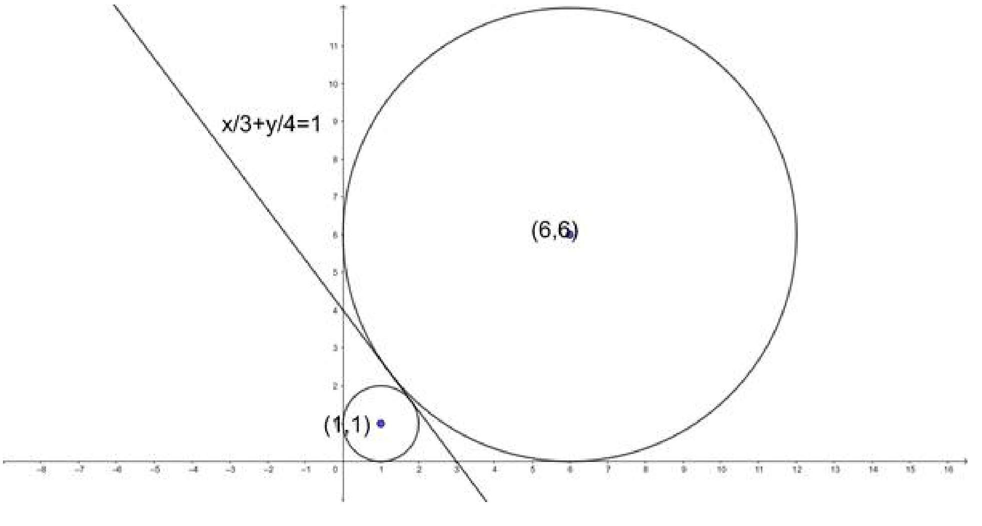
B. 2

C. 3

D. 6

Answer

Given that the equation of the circle which touches the axes of coordinates and the line $\frac{x}{3} + \frac{y}{4} = 1$ and whose centres lie in the first quadrant is $x^2 + y^2 - 2cx - 2cy + c^2 = 0$. We need to find the value of c.



We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

For $x^2 + y^2 - 2cx - 2cy + c^2 = 0$

$\Rightarrow$ Centre(C) = $\left(\frac{-(-2c)}{2}, \frac{-(-2c)}{2}\right)$

$\Rightarrow$ C = (c,c)

$\Rightarrow$ Radius(r) = $\sqrt{c^2 + c^2 - c^2}$

$\Rightarrow r = \sqrt{c^2}$

$\Rightarrow$ r = c

We have $\frac{x}{3} + \frac{y}{4} = 1$ touching the circle.

We know that the perpendicular distance between the circle and centre is equal to the radius of the circle.

We know that the perpendicular distance between from the point (x_1,y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$$\Rightarrow c = \frac{\left|\frac{c}{3} + \frac{c}{4} - 1\right|}{\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2}}$$

$$\Rightarrow c = \frac{\left|\frac{7c}{12} - 1\right|}{\sqrt{\frac{1}{9} + \frac{1}{16}}}.$$

$$\Rightarrow \sqrt{\frac{25}{144}}c = \left|\frac{7c}{12} - 1\right|$$

$$\Rightarrow \left(\sqrt{\frac{25}{144}}c\right)^2 = \left(\left|\frac{7c}{12} - 1\right|\right)^2$$

$$\Rightarrow \frac{25}{144}c^2 = \frac{49}{144}c^2 - \frac{7}{6}c + 1$$

$$\Rightarrow \frac{24}{144}c^2 - \frac{7}{6}c + 1 = 0$$

$$\Rightarrow \frac{1}{6}c^2 - \frac{7}{6}c + 1 = 0$$

$\Rightarrow c^2 - 7c + 6 = 0$

$\Rightarrow c^2 - 6c - c + 6 = 0$

$\Rightarrow$ c(c - 6) - 1(c - 6) = 0

$\Rightarrow$ (c - 1)(c - 6) = 0

$\Rightarrow$ c - 1 = 0 or c - 6 = 0

$\Rightarrow$ c = 1 or c = 6

$\therefore$The correct option is (d).

20. Question

If the circles $x^2 + y^2 = a$ and $x^2 + y^2 - 6x - 8y + 9 = 0$, touch externally, then a =

A. 1

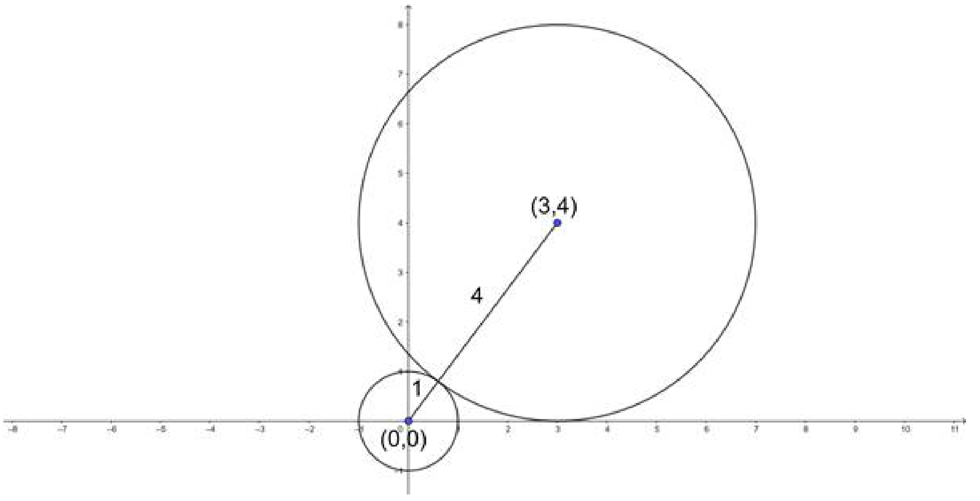
B. - 1

C. 21

D. 16

Answer

Given that the circles $x^2 + y^2 = a$ and $x^2 + y^2 - 6x - 8y + 9 = 0$ touch each other externally.



We need to find the value of a.

We know that if the circles touch each other externally, the distance between the centres is equal to the sum of the radii of two circles.

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

For $x^2 + y^2 = a$

⇒ Centre(C_1) = (0,0)

⇒ Radius(r_1) = $\sqrt{0 + 0 - (-a)}$

⇒ $r_1 = \sqrt{a}$

For $x^2 + y^2 - 6x - 8y + 9 = 0$

⇒ Centre(C_2) = $\left(\frac{-(-6)}{2}, \frac{-(-8)}{2}\right)$

⇒ C_2 = (3,4)

⇒ Radius(r_2) = $\sqrt{(-3)^2 + (-4)^2 - 9}$

⇒ $r_2 = \sqrt{9 + 16 - 9}$

⇒ $r_2 = \sqrt{16}$

⇒ $r_2 = 4$

We have $C_1C_2 = r_1 + r_2$

⇒ $\sqrt{(0-3)^2 + (0-4)^2} = \sqrt{a} + 4$

⇒ $\sqrt{9 + 16} = \sqrt{a} + 4$

⇒ $\sqrt{25} = \sqrt{a} + 4$

⇒ $5 = \sqrt{a} + 4$

⇒ $\sqrt{a} = 5 - 4$

⇒ $\sqrt{a} = 1$

⇒ a = 1

∴The correct option is (a).

21. Question

If (x, 3) and (3, 5) are the extremities of a diameter of a circle with centre at (2, y), then the values of x and y are

A. x = 3, y = 1

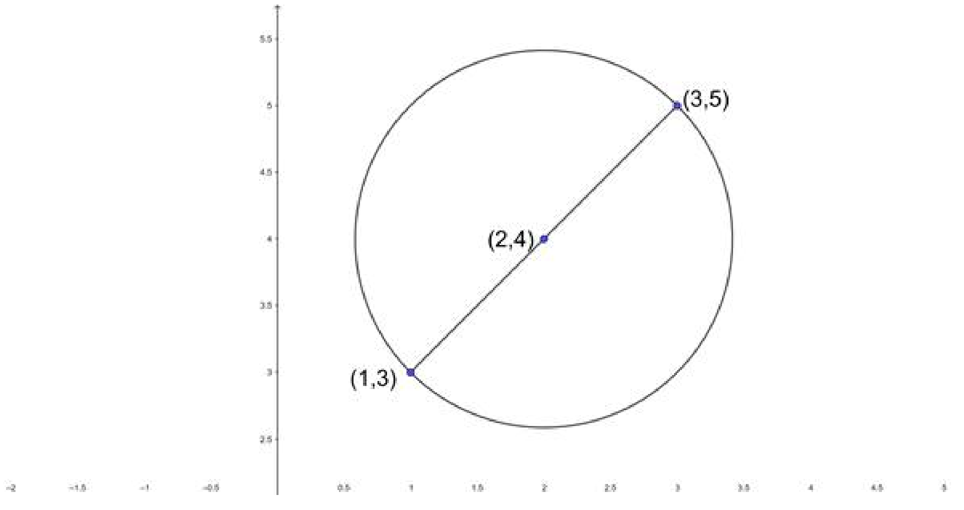
B. x = 4, y = 1

C. x = 8, y = 2

D. none of these

Answer

Given that (x,3) and (3,5) are the extremities of a diameter of a circle with centre (2,y).



We know that centre is the midpoint of diameter.

$\Rightarrow (2,y) = \left(\frac{x+3}{2}, \frac{3+5}{2}\right)$

$\Rightarrow (2,y) = \left(\frac{x+3}{2}, 4\right)$

$\Rightarrow \frac{x+3}{2} = 2 \text{ and } y = 4$

⇒ x + 3 = 4 and y = 4

⇒ x = 1 and y = 4

∴The correct option is (d)

22. Question

If (- 3, 2) lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which is concentric with the circle $x^2 + y^2 + 6x + 8y - 5 = 0$, then c =

A. 11

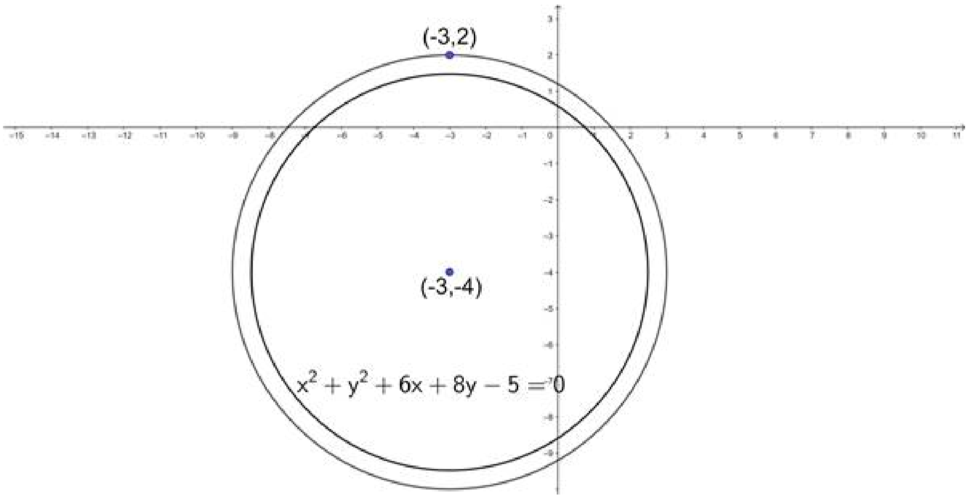
B. - 11

C. 24

D. none of these

Answer

Given that the point (- 3,2) lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which is concentric with $x^2 + y^2 + 6x + 8y - 5 = 0$.



We know that concentric circles will have same centre.

Let us assume the concentric circle be $x^2 + y^2 + 6x + 8y + d = 0$. ..... - (ii)

Substituting (- 3,2) in (ii)

⇒ $(-3)^2 + 2^2 + 6(-3) + 8(2) + d = 0$

⇒ $9 + 4 - 18 + 16 + d = 0$

⇒ $d = -11$

Substituting value of c in (ii) we get

⇒ $x^2 + y^2 + 6x + 8y - 11 = 0$

Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ we get $c = -11$

∴ The correct option is (b).

23. Question

Equation of the diameter of the circle $x^2 + y^2 - 2x + 4y = 0$ which passes through the origin is

A. $x + 2y = 0$

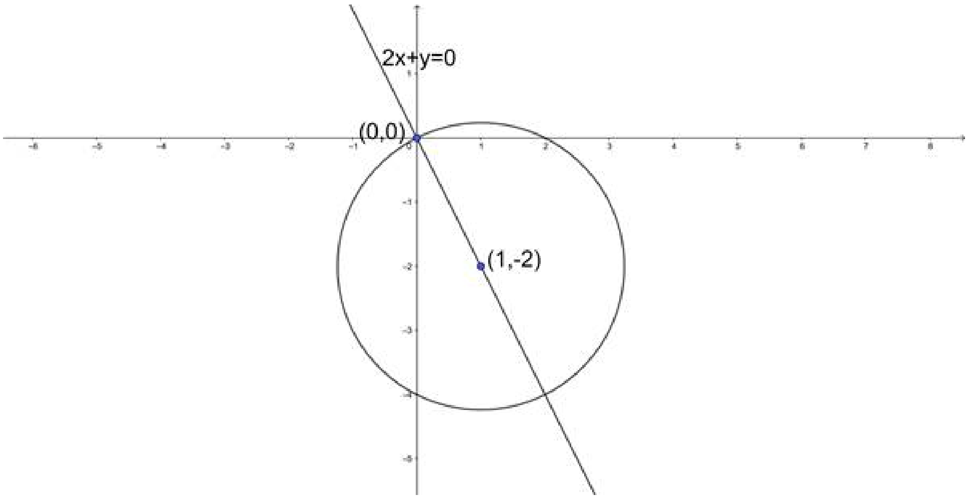
B. $x - 2y = 0$

C. $2x + y = 0$

D. $2x - y = 0$

Answer

Given that we need to find the equation of the diameter if the circle $x^2 + y^2 - 2x + 4y = 0$ which passes through origin.



We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

For $x^2 + y^2 - 2x + 4y = 0$

⇒ Centre(C_1) = $\left(\frac{-(-2)}{2}, \frac{-4}{2}\right)$

⇒ C_1 = (1, - 2)

We know that the diameter of the circle passes through the centre.

We need to find the equation of the diameter passing through the points (1, - 2) and (0,0)

We know that the equation of the straight line passing through the points (x_1,y_1) and (x_2,y_2) is
$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

⇒ $y - 0 = \frac{-2-0}{1-0}(x - 0)$

⇒ $y = \frac{-2}{1}x$

⇒ y = - 2x

⇒ 2x + y = 0

∴The correct option is (c)

24. Question

Equation of the circle through origin which cuts intercepts of length a and b on axes is

A. $x^2 + y^2 + ax + by = 0$

B. $x^2 + y^2 - ax - by = 0$

C. $x^2 + y^2 + bx + ay = 0$

D. none of these

Answer

Given that we need to find the equation of the circle passing through origin and cuts off intercepts a and b from x and y - axes.

Since the circle is having intercept a from x - axis the circle must pass through (a,0) and (- a,0) as it already passes through the origin.

Since the circle is having intercept b from x - axis the circle must pass through (0,b) and (0, - b) as it already passes through the origin.

Let us assume the circle passing through the points O(0,0), A(a,0) and B(0,b).

We know that the standard form of the equation of the circle is given by:

⇒ $x^2 + y^2 + 2fx + 2gy + c = 0$ ..... (1)

Substituting O(0,0) in (1), we get,

⇒ $0^2 + 0^2 + 2f(0) + 2g(0) + c = 0$

⇒ c = 0 ..... (2)

Substituting A(a,0) in (1), we get,

⇒ $a^2 + 0^2 + 2f(a) + 2g(0) + c = 0$

⇒ $a^2 + 2fa + c = 0$ ..... (3)

Substituting B(0,b) in (1), we get,

⇒ $0^2 + b^2 + 2f(0) + 2g(b) + c = 0$

⇒ $b^2 + 2gb + c = 0$ ..... (4)

On solving (2), (3) and (4) we get,

⇒ $f = \frac{-a}{2}, b = \frac{-b}{2}$ and $c = 0$

Substituting these values in (1), we get

⇒ $x^2 + y^2 + 2\left(\frac{-a}{2}\right)x + 2\left(\frac{-b}{2}\right)y + 0 = 0$

⇒ $x^2 + y^2 - ax - by = 0$

Similarly, we get the equation $x^2 + y^2 + ax + by = 0$ for the circle passing through the points (0,0), (- a,0), (0, - b).

∴The equations of the circles are $x^2 + y^2 \pm ax \pm by = 0$.

∴The correct options are (a) and (b).

25. Question

If the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other, then

A. $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c}$

B. $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

C. a + b = 2c

D. $\frac{1}{a}+\frac{1}{b}=\frac{2}{c}$

Answer

Given that the circles $x^2 + y^2 + 2ax + c = 0$ and $x^2 + y^2 + 2by + c = 0$ touch each other externally (assumed).

We need to find the relation of a,b and c.

We know that if the circles touch each other externally, the distance between the centres is equal to the sum of the radii of two circles.

We know that for a circle $x^2 + y^2 + 2ax + 2by + c = 0$

⇒ Centre = (- a, - b)

⇒ Radius = $\sqrt{a^2 + b^2 - c}$

For $x^2 + y^2 + 2ax + c = 0$

⇒ Centre(C_1) = $\left(\frac{-2a}{2}, 0\right)$

⇒ C_1 = (- a,0)

⇒ Radius(r_1) = $\sqrt{(-a)^2 + 0 - c}$

⇒ $r_1 = \sqrt{a^2 - c}$

For $x^2 + y^2 + 2by + c = 0$

$\Rightarrow$ Centre(C_2) = $\left(0, \frac{-2b}{2}\right)$

$\Rightarrow C_2 = (0, -b)$

$\Rightarrow$ Radius(r_2) = $\sqrt{0^2 + (-b)^2 - c}$

$\Rightarrow r_2 = \sqrt{b^2 - c}$

We have $C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{(-a-0)^2 + \left(0-(-b)\right)^2} = \sqrt{a^2 - c} + \sqrt{b^2 - c}$

$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{a^2 - c} + \sqrt{b^2 - c}$

$\Rightarrow \left(\sqrt{a^2 + b^2}\right)^2 = \left(\sqrt{a^2 - c} + \sqrt{b^2 - c}\right)^2$

$\Rightarrow a^2 + b^2 = a^2 - c + b^2 - c + 2\sqrt{(a^2 - c)(b^2 - c)}$

$\Rightarrow 2c = 2\sqrt{(a^2 - c)(b^2 - c)}$

$\Rightarrow c = \sqrt{a^2b^2 - a^2c - b^2c + c^2}$

$\Rightarrow c^2 = a^2b^2 - a^2c - b^2c + c^2$

$\Rightarrow a^2b^2 = a^2c + b^2c$

$\Rightarrow \frac{1}{c} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$

$\Rightarrow \frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}$

$\therefore$The correct option is (a).